(26.55 %, or 23.69 %)-Limiting Highest Efficiencies, obtained respectively in $n^+(p^+) - p(n)$ Crystalline ($X \equiv$ CdTe, or CdSe)-Junction Solar Cells, Due to the Effects of Impurity Size, Temperature, Heavy Doping, and Photovoltaic Conversion

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Abstract:

In the $n^+(p^+) - p(n)$ crystalline ($X \equiv$ CdTe or CdSe)-junction solar cells at 300K, due to the effects of impurity size, temperature, heavy doping, and photovoltaic conversion, we show that, with an increasing donor (acceptor)-radius $r_d(a)$, both the relative dielectric constant and photovoltaic conversion factor decrease, and the intrinsic band gap (IBG) increases, according to the increase in photovoltaic efficiency, as observed in Tables 1-5, being in good accordance with an important result obtained by Shockley and Queisser (1961), stating that for an increasing IBG the photovoltaic efficiency increases.

Further, for highest values of $r_d(a)$, the limiting highest efficiencies are found to be given in Tables 4, 6, as: 26.55 %, and 23.69 %, obtained in such $n^+(p^+) - p(n)$ crystalline (CdTe, or CdSe)-junction solar cells at the open circuit voltage $V_{oc} = 0.82$ V, and 0.89 V, respectively, and at $T=300$ K. Furthermore, from the well-known Carnot-efficiency theorem, as given in Eq. (46), being obtained from the second principle of the thermodynamics, and from the above results of limiting highest efficiencies, the corresponding highest hot reservoir temperatures, $T_H = 408.4$ K, and 393.1 K, respectively. Thus, as noted above, $\eta_{\text{max}}$ and $T_H$ both increase with an increasing IBG, for each ($X \equiv$ CdTe, or CdSe)-crystal at $T=300$ K $\equiv T_C$.

Keywords: donor (acceptor)-size effect, heavily doped emitter region, photovoltaic conversion factor, open circuit voltage, efficiency.

Introduction

In the present work, the limiting highest efficiencies, $\eta_{\text{HID}}$, due to the effects of impurity size, temperature, heavy doping, and photovoltaic conversion, will be investigated respectively in the heavily doped donor (acceptor)-X emitter-and-lightly doped acceptor (donor)-X base-regions, HD[d(a)-X]ER-LD[a(d)-X]BR, $X \equiv$ (CdTe, or CdSe)-crystal, of $n^+(p^+) - p(n)$ junction solar cells; it is inspired from other works (Van Cong, 2022, 2016, 1999, 1995, 1991, 1975; Van Cong et al., 1997, 1996, 1993, 1992, 1990, 1989)
For each d(a)-X-crystal (Van Cong et al., 2022), one notes that: (i) due to the effect of donor (acceptor) d(a)- radius \( r_{d(a)} \), with an increasing \( r_{d(a)} \), the relative dielectric constant \( \varepsilon (r_{d(a)}) \) decreases, while the band gap (BG), \( E_{gn(gp)}(r_{d(a)}) \), increases, as observed in Equations (1.1, 1.2), (ii) due the temperature (T)-effect, the intrinsic BG (IBG), \( E_{gn(gp)}(T, r_{d(a)}) \), decreases with increasing T, as given in Eq. (3), and (iii) due to the effect of heavy doping, with increasing d(a)-density, \( N_{d(a)} \), while the Fermi energy increases, as given in Eq. (5), the BG is reduced by the BG narrowing, as observed in Equations (6) and (7).

Therefore, as remarked in Section 5, the dark carrier-minority saturation current density, \( J_{sc}(t) \), is due to the effect of donor (acceptor)-ionization energy, \( \eta \), with an increasing d(a)-radius.

For each d(a)-X-crystal (Van Cong et al., 2022), one notes that: (i) due to the effect of donor (acceptor) d(a)- radius \( r_{d(a)} \), with an increasing \( r_{d(a)} \), the relative dielectric constant \( \varepsilon (r_{d(a)}) \) decreases, while the band gap (BG), \( E_{gn(gp)}(r_{d(a)}) \), increases, as observed in Equations (1.1, 1.2), (ii) due the temperature (T)-effect, the intrinsic BG (IBG), \( E_{gn(gp)}(T, r_{d(a)}) \), decreases with increasing T, as given in Eq. (3), and (iii) due to the effect of heavy doping, with increasing d(a)-density, \( N_{d(a)} \), while the Fermi energy increases, as given in Eq. (5), the BG is reduced by the BG narrowing, as observed in Equations (6) and (7). Therefore, as remarked in Section 5, the dark carrier-minority saturation current density, \( J_{of(ot)}(W, N_{d(a)}, r_{d(a)}, S; N_{a(d)}, r_{a(d)}; V_{oc}) \), defined in Eq. (37), decreases with increasing IBG, or with increasing \( r_{d(a)} \), given in the ER(BR).

Then, for each d(a)-X-crystal, the photovoltaic conversion effect converts the light, represented by the short circuit current density, defined in Eq. (41), \( J_{sc(t)}(W, N_{d(a)}, r_{d(a)}, S; N_{a(d)}, r_{a(d)}; V_{oc}) \), \( V_{oc} \) being the open circuit voltage, into the electricity, by \( J_{of(ot)}(W, N_{d(a)}, r_{d(a)}, S; N_{a(d)}, r_{a(d)}; V_{oc}) \).

Further, as observed in Tables 3, 4, 5, 6, for a given \( V_{oc} \), the photovoltaic conversion factor (PVCF), determined in Eq. (42), \( \eta_{II}(W, N_{d(a)}, r_{d(a)}, S; N_{a(d)}, r_{a(d)}; V_{oc}) \), decreases with increasing IBG, or with increasing \( r_{d(a)} \), according to the increase in \( J_{sc(t)}(W, N_{d(a)}, r_{d(a)}, S; N_{a(d)}, r_{a(d)}; V_{oc}) \), in the fill factor, \( F_{II}(W, N_{d(a)}, r_{d(a)}, S; N_{a(d)}, r_{a(d)}; V_{oc}) \) and the efficiency \( \eta_{II}(W, N_{d(a)}, r_{d(a)}, S; N_{a(d)}, r_{a(d)}; V_{oc}) \), defined respectively in Equations (43, 44). This remark is found to be in accordance with an important result obtained by Shockley and Queisser (1961), stating that for an increasing IBG, \( \eta \) increases.

**Effects of Impurity Size, Temperature and Heavy Doping**

First of all, in intrinsic X-crystals at temperature \( T(=0 \text{ K}) \), X \( \equiv \) CdTe, or CdSe, and at \( r_{d(a)} = r_{do(ao)} \), the values of relative dielectric constant, \( \varepsilon (r_{d(a)}) = \varepsilon (r_{d(a)} = r_{do(ao)}) = \varepsilon_\alpha \), effective average number of equivalent conduction (valence)-band edges, \( g_{c,v} \), relative effective electron (hole) mass in conduction (valence) bands, \( \left( m_{c,v} / m_0 \right) \), unperturbed intrinsic band gap, \( E_{go} \), and effective ionization energies in absolute values, \( E_{do(ao)} = \frac{13600 \times (m_{c,v} / m_0)}{(\varepsilon_\alpha)^2} \) meV, are given Tables 1 and 2.

**Impurity-Size Effect**

In such d(a)-X systems at T=0 K, since \( r_{d(a)} \), in tetrahedral covalent bonds, is usually either larger or smaller than \( r_{do(ao)} \), a local mechanical strain (or deformation potential energy) is induced, according to a compression (or a dilation), for \( r_{d(a)} > r_{do(ao)} \) (for \( r_{d(a)} < r_{do(ao)} \)), respectively, due to the d(a)-size effect, as that investigated (Van Cong et al., 2022). Further, in n(p)-type X crystals, the band gap \( E_{gn(gp)}(r_{d(a)}) \) and the effective donor (acceptor)-ionization energy \( E_{d(a)}(r_{d(a)}) \) are expressed as:

\[
\text{for } r_{d(a)} = r_{do(ao)} \text{, since } \varepsilon (r_{d(a)}) = \frac{\varepsilon_\alpha}{\sqrt{1 + \left( \frac{r_{d(a)}}{r_{do(ao)}} \right)^3 - 1}} \left| \ln \left( \frac{r_{d(a)}}{r_{do(ao)}} \right) \right| \leq \varepsilon_\alpha,
\]
\[ E_{\text{gn(gp)}}(r_d(a)) - E_{\text{go}} = E_d(a)(r_d(a)) - E_{\text{do}(ao)} = E_{\text{do}(ao)} \times \left[ \left( \frac{\varepsilon_o}{r(r_d(a))} \right)^2 - 1 \right], \quad (1) \]

given the increase in both \( E_{\text{gn(gp)}} \) and \( E_d(a)(r_d(a)) \),
and for \( r_d(a) \leq r_{\text{do}(ao)} \), since 
\[ \varepsilon(r_d(a)) = \varepsilon_{\text{oo}} \times 10^{-4} \times T^2 \]
\[ \left( \frac{r_d(a)}{r_{\text{do}(ao)}} \right)^3 - 1 \times \ln \left( \frac{r_d(a)}{r_{\text{do}(ao)}} \right)^3 \]
\[ \ln \left( \frac{r_d(a)}{r_{\text{do}(ao)}} \right)^3 < 1, \]
\[ E_{\text{gn(gp)}}(r_d(a)) - E_{\text{go}} = E_d(a)(r_d(a)) - E_{\text{do}(ao)} = E_{\text{do}(ao)} \times \left[ \left( \frac{\varepsilon_o}{r(r_d(a))} \right)^2 - 1 \right], \quad (2) \]

corresponding to the decrease in both \( E_{\text{gn(gp)}} \) and \( E_d(a)(r_d(a)) \).

**Temperature Effect**

Here, the intrinsic band gap in the X-crystal is found to be given respectively by (Singh & Ravindra, 2012):
\[ E_{\text{gin(gp)}}(T, r_d(a)) = E_{\text{gn(gp)}}(r_d(a)) - \frac{\alpha \times 10^{-4} \times T^2}{T+\beta}, \quad (3) \]
\[ \alpha = 4.465, 4.3779, \text{ and } \beta = 94 \text{ K, 94 K}. \]

Further, one can define the intrinsic carrier concentration \( n_{\text{in}(ip)} \) by:
\[ n_{\text{in}(ip)}^2(T, r_d(a)) \equiv N_c(T) \times N_v(T) \times \exp \left( \frac{-E_{\text{gin(gp)}}(T, r_d(a))}{k_B T} \right), \quad (4) \]

where \( N_c(T) = 2 \times g_c(T) \times \left( \frac{mc(T)^2 k_B T}{2 \pi \hbar^2} \right)^{\frac{3}{2}} \text{ (cm}^{-3}) \) are the conduction (valence)-band density of states ; \( g_c(T) = 1. \) The numerical results of those energy-band structure parameters are given in the following Tables 1 and 2.

**Table 1.** From Equations (1-4) and in the n(p)-type CdTe crystal, in which \( g_c(T) = 1(1) \) and \( (m_c/m_o) = 0.095 \) and \( (m_v/m_o) = 0.82 \), the numerical results of the energy-band-structure parameters, due to the effect of impurity size are reported, suggesting that, for \( T= 300 \text{ K} \) and
with an increasing \( r_d(a) \), both \( \varepsilon(r_d(a)) \) and \( n_{\text{in}(ip)}(T, r_d(a)) \) decrease, while the other ones increase

<table>
<thead>
<tr>
<th>Donor</th>
<th>As</th>
<th>Te</th>
<th>Sb</th>
<th>Sn</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_d ) (nm)</td>
<td>( \nearrow )</td>
<td>0.118</td>
<td>( r_{\text{do}}=0.132 )</td>
<td>0.136</td>
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<tr>
<td>( \varepsilon(r_d) )</td>
<td>( \nearrow )</td>
<td>10.844</td>
<td>( \varepsilon_o=10.31 )</td>
<td>10.267</td>
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</table>
Table 2. From Equations (1-4) and in the n(p)-type CdSe crystal, in which \( g_{c(v)} = 1(1) \) and \( (m_c/m_a) = 0.11 \) and \( (m_v/m_o) = 0.45 \), the numerical results of the energy-band-structure parameters, due to the effect of impurity size are reported, suggesting that, for \( T= 300 \) K and with an increasing \( r_{d(a)} \), both \( \varepsilon(r_{d(a)}) \) and \( n_{in(ip)}(T, r_{d(a)}) \) decrease, while the other ones increase.

<table>
<thead>
<tr>
<th>Donor</th>
<th>( r_a ) (nm)</th>
<th>( \varepsilon(r_a) )</th>
<th>( E_a(r_a) ) in meV</th>
<th>( E_{gn}(r_a) ) in eV</th>
<th>( E_{gin}(T = 300K, r_a) ) in eV</th>
<th>( n_{in}(T = 300K, r_a) ) in ( 10^{-3} )cm(^{-3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sc</td>
<td>( 0.126 )</td>
<td>11.419</td>
<td>85.5</td>
<td>1.603</td>
<td>1.501</td>
<td>9.225</td>
</tr>
<tr>
<td>Te</td>
<td></td>
<td></td>
<td></td>
<td>0.140</td>
<td>1.517</td>
<td>6.678</td>
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<tr>
<td>Sb</td>
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<td></td>
<td></td>
<td>10.445</td>
<td>1.619</td>
<td>6.424</td>
</tr>
<tr>
<td>Sn</td>
<td></td>
<td></td>
<td></td>
<td>10.344</td>
<td>1.621</td>
<td>6.289</td>
</tr>
<tr>
<td>Cd</td>
<td></td>
<td></td>
<td></td>
<td>10.2</td>
<td>1.52</td>
<td>6.340</td>
</tr>
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<table>
<thead>
<tr>
<th>Acceptor</th>
<th>( n_{io} )</th>
<th>Ga</th>
<th>Mg</th>
<th>In</th>
<th>Cd</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_a ) (nm)</td>
<td>( 0.126 )</td>
<td>0.140</td>
<td>0.144</td>
<td>( r_{ao} = 0.148 )</td>
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</tr>
<tr>
<td>( \varepsilon(r_a) )</td>
<td>11.419</td>
<td>10.445</td>
<td>10.344</td>
<td>( \varepsilon_o = 10.31 )</td>
<td></td>
</tr>
<tr>
<td>( E_a(r_a) ) in meV</td>
<td>85.5</td>
<td>102.2</td>
<td>104.21</td>
<td>( E_{ao} = 104.9 )</td>
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</tr>
<tr>
<td>( E_{gn}(r_a) ) in eV</td>
<td>1.603</td>
<td>1.619</td>
<td>1.621</td>
<td>( E_{go} = 1.622 )</td>
<td></td>
</tr>
<tr>
<td>( E_{gin}(T = 300K, r_a) ) in eV</td>
<td>1.501</td>
<td>1.517</td>
<td>1.519</td>
<td>( E_{gio} = 1.52 )</td>
<td></td>
</tr>
<tr>
<td>( n_{in}(T = 300K, r_a) ) in ( 10^{-3} )cm(^{-3} )</td>
<td>9.225</td>
<td>6.678</td>
<td>6.424</td>
<td>( n_{io} = 6.340 )</td>
<td></td>
</tr>
</tbody>
</table>

Heavy Doping Effect

Here, the Fermi energy \( E_{Fn}(-E_{FP}) \), band gap narrowing (BGN), and apparent band gap narrowing (ABGN), as those determined in P1, are reported in the following.
First, the Fermi energy $E_{Fn}(-E_{FP})$, obtained for any $T$ and any $d(a)$-density, $N_{d(a)}$, being investigated in our previous paper (Van Cong & Debiais, 1993), with a precision of the order of $2.11 \times 10^{-4}$ is found to be given by:

$$
\frac{E_{Fn}(u)}{k_B T} \left( -\frac{E_{FP}(u)}{k_B T} \right) = \frac{G(u) + Au G(u)}{1 + Au}, \quad A = 0.0005372 \quad \text{and} \quad B = 4.82842262,
$$

(5)

where $u$ is the reduced electron density, $u \equiv \frac{N_{d(a)}}{N_{c(v)}}$, $F(u) = au^3 \left( 1 + bu^4 + cu^8 \right)^{2/3}$, $a = \left[ (3\sqrt{\pi}/4) \times u \right]^{2/3}$, $b = \frac{1}{8} \left( \frac{n}{2} \right)^2$, $c = \frac{62.3739855}{1920} \left( \frac{n}{3} \right)^4$, and $G(u) \approx \ln(u) + 2^{-3} \times u \times e^{-du}$, $d = 2^{3/2} \left[ 1 - \frac{3}{16} \right] > 0$.

Here, one notes that: (i) as $u \gg 1$, according to the HD[d(a)-X]ER-case, or to the degenerate case, Eq. (5) is reduced to the function $F(u)$, and (ii) $\frac{E_{Fn}(u<1)}{k_B T} \left( -\frac{E_{FP}(u<1)}{k_B T} \right) \ll -1$, to the LD[a(d)-X]BR-case, or to the non-degenerate case, Eq. (5) is reduced to the function $G(u)$.

Secondly, if denoting the effective Wigner-Seitz radius $r_s$, characteristic of the interactions, by:

$$
r_{sn(sp)}(N_{d(a)}, r_{d(a)}) = 1.1723 \times 10^8 \times \left( \frac{g_{c(v)}}{N_{d(a)}} \right)^{1/3} \times \frac{m_{c(v)}}{\epsilon(r_{d(a)})},
$$

the correlation energy of an effective electron gas, $E_{cn(cp)}(N_{d(a)}, r_{d(a)})$, is given as (Van Cong et al., 2022):

$$
E_{cn(cp)}(N_{d(a)}, r_{d(a)}) = \frac{-0.87553 + 0.093280}{0.0908 + r_{sn(sp)}} + \frac{0.87553 + \frac{2[1+\ln(2)]}{\pi^2} \times \ln(4) \times r_{sn(sp)}}{1 + 0.03847728 \times 1.67378876 \times r_{sn(sp)}^2}.
$$

Now, taking into account various spin-polarized chemical potential-energy contributions such as (Van Cong, 2016): exchange energy of an effective electron (hole) gas, majority-carrier correlation energy of an effective electron (hole) gas, minority hole (electron) correlation energy, majority electron (hole)-ionized $d(a)$ interaction screened Coulomb potential energy, and finally minority hole (electron)-ionized $d(a)$ interaction screened Coulomb potential energy, the band gap narrowing (BGN) are given as follows.

Then, in the n-type heavily doped X-crystals, the BGN is found to be given by:

$$
\Delta E_{gn}(N_d, r_d) \approx a_1 \times \frac{\epsilon}{\epsilon(r_d)} \times N_r^{1/3} + a_2 \times \frac{\epsilon}{\epsilon(r_d)} \times N_r^{1/3} \times (2.503 \times [-E_c(r_{sn}) \times r_{sn}]) + a_3 \times \left[ \frac{\epsilon}{\epsilon(r_d)} \right]^{5/4} \times \frac{m_v}{m_e} \times N_r^{1/4} + a_4 \times \frac{\epsilon}{\epsilon(r_d)} \times N_r^{1/2} \times 2 + a_5 \times \left[ \frac{\epsilon}{\epsilon(r_d)} \right]^{3/2} \times N_r^{1/2} \times \left[ \frac{\epsilon}{\epsilon(r_d)} \right]^{3/2} \times N_r^{1/2}, \quad N_r \equiv \left( \frac{N_d}{9.999 \times 10^{17} \text{cm}^{-3}} \right),
$$

(6)
where $a_1 = 3.8 \times 10^{-3}$ (eV), $a_2 = 6.5 \times 10^{-4}$ (eV), $a_3 = 2.8 \times 10^{-3}$ (eV), $a_4 = 5.597 \times 10^{-3}$ (eV) and $a_5 = 8.1 \times 10^{-4}$ (eV), and in the p-type heavily doped X-crystals, as:

$$
\Delta E_{gp}(N_a, r_a) \approx a_1 \times \left( \frac{e_0}{\varepsilon_0(r_a)} \right)^{5/4} + a_2 \times \left( \frac{e_0}{\varepsilon_0(r_a)} \right)^{1/2} + a_3 \times \left[ \frac{e_0}{\varepsilon_0(r_a)} \right]^{3/2} + a_4 \times \left( \frac{e_0}{\varepsilon_0(r_a)} \right)^{1/3} + a_5 \times \left( \frac{e_0}{\varepsilon_0(r_a)} \right)^{1/2},
$$

(7)

where $a_1 = 3.15 \times 10^{-3}$ (eV), $a_2 = 5.41 \times 10^{-4}$ (eV), $a_3 = 2.32 \times 10^{-3}$ (eV), $a_4 = 4.12 \times 10^{-3}$ (eV) and $a_5 = 9.80 \times 10^{-5}$ (eV).

Therefore, in the HD[d(a)-X]ER, we can define the effective extrinsic carrier concentration, $n_{en}^*(ep)$, by:

$$
n_{en}^*(ep) (N_d(a), T, r_d(a)) \equiv \sqrt{N_d(a) \times p_0(n_o)} = n_{in}(ip) \times \exp \left[ \frac{\Delta E_{agn}(agp)}{2k_BT} \right],
$$

(8)

where the apparent band gap narrowing (ABGN), $\Delta E_{agn}(agp)$, is defined by:

$$
\Delta E_{agn}(N_d, T, r_d) \equiv \Delta E_{gn} + k_BT \times \ln \left( \frac{N_d}{N_e} \right) - E_{Fn}(N_d, T),
$$

$$
\Delta E_{agp}(N_a, T, r_a) \equiv \Delta E_{gp} + k_BT \times \ln \left( \frac{N_a}{N_v} \right) + E_{Fp}(N_a, T). \tag{9}
$$

**Total minority-carrier saturation current density**

In the two $n^+(p^+) - p(n)$ X-junction solar cells, denoted respectively by I(II), the total carrier-minority saturation current density is defined by:

$$
J_{oI(oII)} \equiv J_{Eno(Epo)} + J_{Bpo(Bno)} \tag{10}
$$

where $J_{Bpo(Bno)}$ is the minority-electron (hole) saturation current density injected into the LD[a(d)-X]BR, and $J_{Eno(Epo)}$ is the minority-hole (electron) saturation-current density injected into the HD[d(a)-X]ER.

**$J_{Bpo(Bno)}$ in the LD[a(d)-X]BR**

Here, $J_{Bpo(Bno)}$ is determined by (Van Cong, 1999):

$$
J_{Bpo(Bno)}(N_a(d), r_a(d)) = \frac{e \times n_{ip(in)}^2(r_a(d)) \times \sqrt{D_{e(h)}(N_a(d), r_a(d))}}{N_a(d)},
$$

(11)
where \( n_{ip}^2 \) is determined in Tables 1 and 2, \( D_e(N_a, r_a) \) is the minority electron (minority hole) diffusion coefficient:

\[
D_e(N_a, r_a) = \frac{kT}{e} \times \left[ 850 + \frac{5750}{1 + \left( \frac{N_a}{6 \times 10^{16}\, \text{cm}^{-3}} \right)^{1.2}} \right] \times \left( \frac{r_a}{e_0} \right)^2 \, \text{(cm}^2\text{s}^{-1})
\]

(12)

\[
D_h(N_d, r_d) = \frac{kT}{e} \times \left[ 85 + \frac{1165}{1 + \left( \frac{N_d}{4 \times 10^{17}\, \text{cm}^{-3}} \right)^{0.62}} \right] \times \left( \frac{r_d}{e_0} \right)^2 \, \text{(cm}^2\text{s}^{-1})
\]

(13)

and \( \tau_{eB(hB)}(N_a) \) is the minority electron (minority hole) lifetime in the BR:

\[
\tau_{eB}(N_a)^{-1} = \frac{1}{10^{-7}} + 3 \times 10^{-13} \times N_a + 1.83 \times 10^{-31} \times N_a^2
\]

(14)

\[
\tau_{hB}(N_d)^{-1} = \frac{1}{10^{-7}} + 11.76 \times 10^{-13} \times N_d + 2.78 \times 10^{-31} \times N_d^2
\]

(15)

**J_{Eno(Epo)} in the HD[d(a)-X]ER**

In the non-uniformly and heavily doped emitter region of d(a)-X devices, the effective Gaussian d(a)-density profile or the d(a) (majority-e(h)) density, is defined in such the HD[d(a)-X]ER-width W, as (Van Cong et al., 2022):

\[
\rho_{d(a)}(x, N_{d(a)}, W) = N_{d(a)} \times \exp \left\{ -\left( \frac{x}{W} \right)^2 \times \ln \left( \frac{N_{d(a)}}{N_{do(ao)(W)}} \right) \right\} \equiv N_{d(a)} \times \left[ \frac{N_{d(a)}}{N_{do(ao)(W)}} \right]^{-\left( \frac{x}{W} \right)^2}, \quad 0 \leq x \leq W,
\]

\[
N_{do(ao)(W)} \equiv 7.9 \times 10^{17} \times (2 \times 10^5) \times \exp \left\{ -\left( \frac{W}{184.2 \times 10^{-7}\, \text{cm}} \right)^{1.066(0.5)} \right\} \, \text{(cm}^{-3})
\]

(16)

where \( \rho_{d(a)}(x = 0) = N_{d(a)} \) is the surface d(a)-density, and at the emitter-base junction, \( \rho_{d(a)}(x = W) = N_{do(ao)(W)} \), which decreases with increasing W. Further, the “effective doping density” is defined by:

\[
N_{d(a)}^* (x, r_{d(a)}) \equiv \frac{\rho_{d(a)}(x)}{\exp \left\{ \frac{\Delta E_{agn}(agp)(\rho_{d(a)}r_{d(a)})}{kT} \right\}}
\]

\[
N_{d(a)}^* (x = 0, r_{d(a)}) \equiv \frac{N_{d(a)}}{\exp \left\{ \frac{\Delta E_{agn}(agp)(N_{d(a)}r_{d(a)})}{kT} \right\}}
\]

and

\[
N_{d(a)}^* (x = W, r_{d(a)}) \equiv \frac{N_{do(ao)(W)}}{\exp \left\{ \frac{\Delta E_{agn}(agp)(N_{do(ao)(W)}r_{d(a)})}{kT} \right\}}
\]

(17)
where the apparent band gap narrowing $\Delta E_{\text{agn}}$ is determined in Eq. (9), replacing $N_{d(a)}$ by $\rho_{d(a)}(x, N_{d(a)}, W)$. The same remark can be applied to following Equations (18-20).

Now, we can define the minority hole (minority electron) transport parameter $F_{h(e)}$ as:

$$F_{h(e)}(N_{d(a)}, r_{d(a)}) \equiv \frac{n_{\text{in(ip)}}(T, r_{d(a)})}{p_0(n_o) \times D_{h(e)}} = \frac{N_{d(a)}}{D_{h(e)}} \times \left(\frac{n_{\text{in(ip)}}}{n_{\text{in(ip)}}} \right)^2 \equiv \frac{N_{d(a)}}{D_{h(e)} \times \exp\left(\frac{\Delta E_{\text{agn}}}{k_B T}\right)} \text{ (cm}^{-5} \times \text{s)},$$

(18)

the minority hole (electron) diffusion length, $L_{h(e)}(N_{d(a)}, r_{d(a)})$ by:

$$L_{h(e)}^{-2}(N_{d(a)}, r_{d(a)}) = \left[\tau_{hE(e)} \times D_{h(e)}\right]^{-1} = \left(C \times F_{h(e)} \right)^2 = \left(C \times \frac{N_{d(a)}}{D_{h(e)}} \right)^2 = \left(C \times \frac{n_{\text{in(ip)}}(r_{d(a)})}{p_0(n_o) \times D_{h(e)}} \right)^2,$$

(19)

where the constant $C$ was chosen to be equal to: $2.0893 \times 10^{-30}$ (cm$^4$/s), and the minority hole (minority electron) lifetime $\tau_{hE(e)}$ as:

$$\tau_{hE(e)} \equiv \frac{1}{D_{h(e)} \times L_{h(e)}^{-2}} = \frac{1}{D_{h(e)} \times \left(C \times F_{h(e)} \right)^2}.$$  

(20)

Then, under low-level injection, in the absence of external generation, and for the steady-state case, we can define the minority-h(e) density by:

$$p_0(x)[n_o(x)] \equiv \frac{n_{\text{in(ip)}}}{N_{d(a)}(x, r_{d(a)})},$$

(21)

and a normalized excess minority-h(e) density $u(x)$ or a relative deviation between $p(x)[n(x)]$ and $p_0(x)[n_o(x)]$.

$$u(x) \equiv \frac{p(x)[n(x)] - p_0(x)[n_o(x)]}{p_0(x)[n_o(x)]},$$

(22)

which must verify the two following boundary conditions as:

$$u(x = 0) \equiv \frac{-J_h(x=0)[I_e(x=0)]}{eS \times p_0(x=0)[n_o(x=0)]},$$

$$u(x = W) = \exp\left(\frac{V}{n_{i(1)}(V) \times V_T}\right) - 1.$$
Here, $n_{I(W)}(V)$ is a photovoltaic conversion factor determined latter, $S \left( \frac{cm}{s} \right)$ is the surface recombination velocity at the emitter contact, $V$ is the applied voltage, $V_T \equiv (k_B T / e)$ is the thermal voltage, and the minority-hole (electron) current density $J_h(x, r_{d(a)})$.

Further, from the Fick’s law for minority hole (electron)-diffusion equations, one has (Van Cong et al., 2022):

$$J_h(x, r_{d(a)}) = \frac{-e(e+) \times n_{I(n)(p)}^2}{F_h(x)} \times \frac{du(x)}{dx} = \frac{-e(e+) \times n_{I(n)(p)}^2 D_h(x) N_{d(a)} r_{d(a)}}{N_{d(a)}^2 (x, r_{d(a)})} x \frac{du(x)}{dx},$$

where $N_{d(a)}^*(x, r_{d(a)})$ is given in Eq. (17), $D_h(x)$ and $F_h(x)$ are determined respectively in Equations (12, 13, 18), and from the minority-hole (electron) continuity equation as:

$$
\frac{d J_h(x, r_{d(a)})}{dx} = -e(e+) \times n_{I(n)(p)}^2 \times \frac{u(x)}{F_h(x) \times L_h(x)} = -e(e+) \times n_{I(n)(p)}^2 \times \frac{u(x)}{N_{d(a)}^* (x, r_{d(a)}) \times \tau_{hE}(x)}.
$$

Therefore, the following second-order differential equation is obtained:

$$
\frac{d^2 u(x)}{dx^2} - \frac{d F_h(x)}{dx} x \frac{du(x)}{dx} - \frac{u(x)}{L_h(x)} = 0,
$$

Then, taking into account the two above boundary conditions given in Eq. (22), one thus gets the general solution of this Eq. (25), as:

$$u(x) = \frac{\sinh(P(x)) + I(W, S) \times \cosh(P(x))}{\sinh(P(W)) + I(W, S) \times \cosh(P(W))} \times \left( \exp \left( \frac{V}{n_{I(W)}(V) \times V_T} \right) - 1 \right),$$

where the factor $I(W, S)$ is determined by:

$$I(W, S) = \frac{D_h((W) \times L_h((W))}{S \times L_h((W))}. \quad (27)$$

Further, since $dP(x) = C x = 1 \frac{C}{L_h(x)^3}$, $C = 2.0893 \times 10^{-30}$ (cm$^4$/s), for the X-crystal, being an empirical parameter, chosen for each crystalline semiconductor, $P(x)$ is thus found to be defined by:

$$P(x) = \int_0^x \frac{dx}{L_h(x)}, \quad 0 \leq x \leq W, \quad P(x = W) = \left( \frac{1}{W} \times \int_0^W \frac{dx}{L_h(x)} \right) \times W = \frac{W}{L_h(x)} \times L_h(x) = \frac{W}{L_h(x)} \times L_h(x) = \frac{W}{L_h(x)} \times L_h(x), \quad (28)$$
where $L^*_K(x)$ is the effective minority hole (minority electron) diffusion length. Further, the minority-hole (electron) current density injected into the HD[d(a)-X]ER is found to be given by:

$$J_h(x, W, N_d(a), r_{d(a)}, S, V) = -J_{Eno}(x, W, N_d, r_d, S) [J_{Epo}(x, W, N_d, r_d, S)] \times \left( \exp \left( \frac{V}{n_{(H)}(V) \times V_T} \right) - 1 \right),$$

(29)

where $J_{Eno(Epo)}$ is the saturation minority hole (minority electron) current density,

$$J_{Eno(Epo)}(x, W, N_d(a), r_{d(a)}, S) = \frac{e \tau_{h(Ep)} \times D_h}{N^*_d(x, r_{d(a)} \times L_{h(e)}} \times \frac{\cosh(P(x)) + I(W, S) \times \sinh(P(x))}{\sinh(P(W)) + I(W, S) \times \cosh(P(W))}.$$  

(30)

In the following, we will denote $P(W)$ and $I(W, S)$ by $P$ and $I$, for a simplicity. So, Eq. (30) gives:

$$J_{Eno(Epo)}(x = 0, W, N_d(a), r_{d(a)}, S) = \frac{e \tau_{h(Ep)} \times D_h}{N^*_d(x, r_{d(a)} \times L_{h(e)}} \times \frac{1}{\sinh(P) + 1 \times \cosh(P)},$$

(31)

$$J_{Eno(Epo)}(x = W, W, N_d(a), r_{d(a)}, S) = \frac{e \tau_{h(Ep)} \times D_h}{N^*_d(x = W, r_{d(a)} \times L_{h(e)}} \times \frac{\cosh(P) + I \times \sinh(P)}{\sinh(P) + I \times \cosh(P)},$$

(32)

and then,

$$J_{h(e)}(x = 0, W, N_d(a), r_{d(a)}, S, V) \equiv J_{Eno(Epo)}(x = 0, W, N_d(a), r_{d(a)}, S) = \frac{1}{\cosh(P) + I \times \sinh(P)}.$$  

(33)

Now, if defining the effective excess minority-hole (electron) charge storage in the emitter region by:

$$Q^*_h(x = W, N_d(a), r_{d(a)}) \equiv \int_0^W e(-e) \times u(x) \times p_o(x)[n_o(x)] \times \frac{\tau_{h(Ep)}(N_d(a), r_{d(a)})}{\tau_h(Ep)(r_{d(a)})} dx,$$

and the effective minority hole (minority electron) transit time [htt(eet)] by:

$$\tau^{*}_{htt(eet)}(x = W, W, N_d(a), r_{d(a)}, S) \equiv Q^*_h(x = W, N_d(a), r_{d(a)}) / J_{Eno(Epo)}(x = W, W, N_d(a), r_{d(a)}, S),$$

and from Equations (24, 31), one obtains:

$$\frac{\tau^{*}_{htt(eet)}(x = W, W, N_d(a), r_{d(a)}, S)}{\tau_h(Ep)} = 1 - \frac{J_{Eno(Epo)}(x = 0, W, N_d(a), r_{d(a)}, S)}{J_{Eno(Epo)}(x = W, W, N_d(a), r_{d(a)}, S)} \equiv 1 - \frac{1}{\cosh(P) + I \times \sinh(P)}.$$  

(34)

Now, some important results can be obtained and discussed below.
As \( P \ll 1 \) (or \( W \ll L_{\text{h}(e)} \)) and \( S \to \infty \), \( I \equiv I(W,S) = \frac{D_{\text{h}(e)}(N_{\text{do}(a)}(W))}{S \times L_{\text{h}(e)}(N_{\text{do}(a)}(W))} \to 0 \), from Eq. (34), one has:

\[
\frac{\tau_{\text{h},(\text{E})}^*(x=W,W,N_{d(a)},r_{d(a)},S)}{\tau_{\text{h},(\text{E})}} \to 0, \text{ suggesting a completely transparent emitter region (CTER)-case, where, from Eq. (32), one obtains:}
\]

\[
J_{\text{En}o(\text{Epo})}(x = W, N_{d(a)}, r_{d(a)}, S \to \infty) \to \frac{e n_{\text{in}(\text{ip})}^2 D_{\text{h}(e)}}{N_{d(a)}^*(x=W,r_{d(a)} \times L_{\text{h}(e)}(W)} \times \frac{1}{P(W)}
\]

Further, as \( P \gg 1 \) (or \( W \gg L_{\text{h}(e)} \)) and \( S \to 0 \), \( I \equiv I(W,S) = \frac{D_{\text{h}(e)}(N_{\text{do}(a)}(W))}{S \times L_{\text{h}(e)}(N_{\text{do}(a)}(W))} \to \infty \), and from Eq. (34) one has:

\[
\frac{\tau_{\text{h},(\text{E})}^*(x=W,W,N_{d(a)},r_{d(a)},S)}{\tau_{\text{h},(\text{E})}} \to 1, \text{ suggesting a completely opaque emitter region (COER)-case, where, from Eq. (32), one gets:}
\]

\[
J_{\text{En}o(\text{Epo})}(x = W, N_{d(a)}, r_{d(a)}, S \to 0) \to \frac{e n_{\text{in}(\text{ip})}^2 D_{\text{h}(e)}}{N_{d(a)}^*(x=W,r_{d(a)} \times L_{\text{h}(e)}(W)} \times \tanh(P).
\]

In summary, in the two \( n^+(p^+) - p(n) \) X-junction solar cells, the dark carrier-minority saturation current density \( J_{\text{ol}(\text{off})} \), defined in Eq. (10), is now rewritten as:

\[
J_{\text{ol}(\text{off})}(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r'_{a(d)}) \equiv J_{\text{En}o(\text{Epo})}(W, N_{d(a)}, r_{d(a)}, S) + J_{\text{Bpo}(\text{Bno})}(N_{a(d)}, r'_{a(d)}),
\]

where \( J_{\text{En}o(\text{Epo})} \) and \( J_{\text{Bpo}(\text{Bno})} \) are determined respectively in Equations (32, 11).

**Photovoltaic conversion effect at 300K**

Here, in the \( n^+(p^+) - p(n) \) X-junction solar cells at \( T=300 \) K, denoted respectively by I(II), and for physical conditions, respectively, as:

\[
W = 0.1 \ (0.1) \ \mu m, \ N_{d \equiv T(e)(a \equiv C d)} = 10^{19} \ (10^{20}) \ cm^{-3}, r_{d(a)}(W), S = \\
100 \ (100) \ \left(\frac{cm^3}{s}\right); \ N_{a \equiv C d(d \equiv T e)} = 10^{17}(10^{17}) \ cm^{-3}, r_{a(a)}(d), \text{ for } X \equiv C d T e, \text{ and}
\]

\[
W = 0.1 \ (0.1) \ \mu m, \ N_{d \equiv T(e)(a \equiv C d)} = 10^{19} \ (10^{20}) \ cm^{-3}, r_{d(a)}(W), S = \\
100 \ (100) \ \left(\frac{cm^3}{s}\right); \ N_{a \equiv C d(d \equiv T e)} = 10^{17}(10^{17}) \ cm^{-3}, r_{a(a)}(d), \text{ for } X \equiv C d S e,
\]

we propose, at given open circuit voltages: \( V_{\text{oc}1(\text{oc}2)} \) and \( V_{\text{oc}2(\text{oc}1)} \), the corresponding data of the short circuit current density \( J_{\text{sc}(\text{l}1)} \), in order to formulate our following treatment method of two fixe points as:
for $X \equiv CdTe$ and at $V_{oc1(oct)} = V_{oc11(oct)} = 0.73 \ (0.8759) \ V$,
\[ J_{sc1(oct)} = \frac{0.0216 \ (0.03025)}{A/cm^2}, \text{ and} \]
for $X \equiv CdSe$ and at $V_{oc1(oct)} = V_{oc11(oct)} = 0.87 \ (1.03) \ V$,
\[ J_{sc1(oct)} = \frac{0.02432 \ (0.0202)}{A/cm^2}. \]

(39.1)

(39.2)

Now, we define the net current density $J$ at $T=300 \ K$, obtained for the infinite shunt resistance, and expressed as a function of the applied voltage $V$, flowing through the $n^+(p^+) - p(n)$ X-junction of solar cells, as:

\[ J(V) \equiv J_{ph}(V) - J_{ol(oct)} \times \left( e^{X_{I(I)}(V)} - 1 \right), \]

where $X_{I(I)}$ is the photocurrent density defined by:

\[ J_{ph}(V) = \frac{V}{n_{I(I)}(V) \times V_T}, \quad \text{and} \quad V_T \equiv \frac{k T}{e} = 0.02585 \ V. \]

(40)

where the function $n_{I(I)}(V)$ is the photovoltaic conversion factor (PVCF), noting that as $V = V_{oc}$, being the open circuit voltage, $J(V) = 0$, the photocurrent density is defined by:

\[ J_{sc1(oct)}(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}) \equiv J_{ol(oct)}(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}) \times \left( e^{X_{I(I)}(V_{oc})} - 1 \right), \]

(41)

where $n_{I(I)}(V_{oc}) = n_{I(I)}(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc})$, and $X_{I(I)}(V_{oc}) = \frac{V_{oc}}{n_{I(I)}(V_{oc}) \times V_T}$.

Here, one remarks that (i) for a given $V_{oc}$, both $n_{I(I)}$ and $J_{ol(I)}$ have the same variations, obtained in the same physical conditions, as observed in many cases, and (ii) the function $(e^{X_{I(I)}(V_{oc})} - 1)$ or the PVCF, $n_{I(I)}$, representing the photovoltaic conversion effect, converts the light, represented by $J_{sc1(oct)}$, into the electricity, by $J_{ol(oct)}$.

Then, from Eq. (41), for $n^+(p^+) - p(n)$ X-junction solar cells, one respectively obtained:

for $X \equiv CdTe$,

\[ n_{I(1)(2)}(V_{oc1(oct)}, J_{sc1(oct)}) = 0.6846 \ (0.8147) \text{ and } n_{I(1)(2)}(V_{oc11(oct)}, J_{sc11(oct)}) = 0.70157 \ (0.8348), \text{ and for } X \equiv CdSe, \]

\[ n_{I(1)(2)}(V_{oc1(oct)}, J_{sc1(oct)}) = 0.6656 \ (0.7910) \text{ and } n_{I(1)(2)}(V_{oc11(oct)}, J_{sc11(oct)}) = 0.6618 \ (0.7864). \]
Now, for $V_{oc} \geq V_{oc1(oc11)}$ and $X \equiv (\text{CdTe}, \text{ or } \text{CdSe})$, respectively, one can propose the general expressions for the PVCF, in order to get exactly the above obtained values of $n_{1(II2)}$ and $n_{II1(II2)}$,

$$n_{II}(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}) = n_{1II1} + n_{II2} \times \left(\frac{V_{oc}}{V_{oc1(ocII1)}} - 1 \right)^{\alpha(\beta)},$$  \hspace{1cm} (42)

where $\alpha(\beta) = 1.139 \ (1.13972), \ 1.0878 \ (1.0881)$.

Therefore, one can determine the general expressions for the fill factors, as:

$$F_{II}(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V) = \frac{x_{II}(V_{oc}) - \ln[x_{II}(V_{oc}) + b]}{x_{II}(V_{oc}) + a}, \quad a, b \geq 0,$$  \hspace{1cm} (43)

where $a$ and $b$ will be chosen here, as: $a=1, b=0.72$, corresponding to the ideal $X$-junction solar cells, and $a=b=0$, according to limiting highest values of $F_{II}$.

Finally, the efficiency $\eta_{II}$ can be defined in the $n^+(p^+) - p(n)$ junction solar cells, by:

$$\eta_{II}(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}) \equiv \frac{I_{sc(lt,lt)} \times V_{oc} \times F_{II}}{P_{in}},$$  \hspace{1cm} (44)

being assumed to be obtained at 1 sun illumination or at AM1.5G spectrum ($P_{in} = 0.100 \ \text{W/cm}^2$).

**Numerical Results and Concluding Remarks**

We will respectively consider the two following cases:

HD [P(B); As (Ga); Sb(Mg); Sn(In)] X – ER – LD [B (P); Ga (As); Mg (Sb); In (Sn)] X – BR – cases,

noting that, for each $X$-crystal, one has: 4 ($n^+p$) – junctions: $(P^+B, As^+Ga, Sb^+Mg, Sn^+In)$, and 4 ($p^+n$) – junctions: $(B^+P, Ga^+As, Mg^+Sb, In^+Sn)$. Then, one proposes the physical conditions for the determination of various photovoltaic conversion coefficients, as:

$$W = 0.1 \ (0.1) \ \text{µm}, \quad N_{d(a)} = 10^{19} \ (10^{20}) \ \text{cm}^{-3}, \quad S = 100 \ (100) \ \text{(cm/s)}; \quad N_{a(d)} = 10^{17} \ (10^{17}) \ \text{cm}^{-3}, \quad \text{for } X \equiv \text{CdTe}, \text{ and}\ (45.1)$$

$$W = 0.1 \ (0.1) \ \text{µm}, \quad N_{d(a)} = 10^{19} \ (10^{20}) \ \text{cm}^{-3}, \quad S = 100 \ (100) \ \text{(cm/s)}; \quad N_{a(d)} = 10^{17} \ (10^{17}) \ \text{cm}^{-3}, \quad \text{for } X \equiv \text{CdSe}. \hspace{1cm} (45.2)$$

**$n^+(p^+) - p(n)$ CdTe-Junction Solar Cells**

**First HD [As; Te; Sb; Sn] CdTe – ER – LD [Ga; Mg; In; Cd] CdTe – BR – case**

Here, there are the 4 ($n^+p$) – CdTe junctions, being denoted by: $(As^+Ga, Te^+Mg, Sb^+In, Sn^+Cd)$. 

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Then, from the physical conditions given in Eq. (45.1), one respectively gets: from Eq. (34), \( \frac{\tau_{nt}}{\tau_{hE}} = (0, 0, 0, 0) \), suggesting a completely transparent condition, from Eq. (32), \( J_{Eno} = (2.42, 2.11, 2.02, 1.99) \times 10^{-23} \left( \frac{A}{cm^2} \right) \), from Eq. (11), \( J_{Bpo} = (6.19, 2.97, 2.72, 2.64) \times 10^{-20} \left( \frac{A}{cm^2} \right) \approx J_{Bpo} \), which decreases with an increasing \( r_a \), due to the increase in the IBG, as observed in Table 1.

**Second HD [Ga; Mg; In; Cd] CdTe – ER – LD [As; Te; Sb; Sn] CdTe – BR – case**

Here, there are 4 \( (p^+ n) – CdTe \) junctions: \( (Ga^+ As, Mg^+ Te, In^+ Sb, Cd^+ Sn) \).

Then, from the physical conditions given in Eq. (45.1), one respectively obtains: from Eq. (34), \( \frac{\tau_{nt}}{\tau_{hE}} = (0, 0, 0, 0) \) suggesting a completely transparent condition, from Eq. (32), \( J_{Eno} = (2.24, 2.21, 2.20, 2.19) \times 10^{-23} \left( \frac{A}{cm^2} \right) \), from Eq. (11), \( J_{Bpo} = (6.19, 2.97, 2.72, 2.64) \times 10^{-20} \left( \frac{A}{cm^2} \right) \approx J_{Bpo} \), which decreases with an increasing \( r_a \), due to the increase in the IBG, as observed in Table 1.

**n+ \( (p^+) – p \ (n) \) CdSe-Junction Solar Cells**

**First HD [Se; Te; Sb; Sn] CdSe – ER – LD [Ga; Mg; In; Cd] CdSe – BR – case**

Here, there are the 4 \( (n^+ p) – CdSe \)-junctions, being denoted by: \( (Se^+ Ga, Te^+ Mg, Sb^+ In, Sn^+ Cd) \).

Then, from the physical conditions given in Eq. (45.2), one respectively gets: from Eq. (34), \( \frac{\tau_{nt}}{\tau_{hE}} = (0, 0, 0, 0) \) suggesting a completely transparent condition, from Eq. (32), \( J_{Eno} = (1.33, 0.69, 0.64, 0.62) \times 10^{-27} \left( \frac{A}{cm^2} \right) \), from Eq. (11), \( J_{Bno} = (1.05, 0.95, 0.94, 0.92) \times 10^{-20} \left( \frac{A}{cm^2} \right) \), and finally from Eq. (37), \( J_{oI} = (2.18, 0.79, 0.73, 0.71) \times 10^{-27} \left( \frac{A}{cm^2} \right) \approx J_{Epo} \), which decreases with an increasing \( r_a \), due to the increase in the IBG, as observed in Table 2.

**Second HD [Ga; Mg; In; Cd] CdSe ER – LD [Se; Te; Sb; Sn] CdSe BR – case**

Here, there are 4 \( (p^+ n) – CdSe \) junctions: \( (Ga^+ Se, Mg^+ Te, In^+ Sb, Cd^+ Sn) \).

Then, from the physical conditions given in Eq. (45.2), one respectively obtains: from Eq. (34), \( \frac{\tau_{nt}}{\tau_{hE}} = (0, 0, 0, 0) \) suggesting a completely transparent condition, from Eq. (32), \( J_{Eno} = (1.33, 0.69, 0.64, 0.62) \times 10^{-27} \left( \frac{A}{cm^2} \right) \), from Eq. (11), \( J_{Bno} = (1.33, 0.69, 0.64, 0.62) \times 10^{-27} \left( \frac{A}{cm^2} \right) \approx J_{Epo} \), which decreases with an increasing \( r_a \), due to the increase in the IBG, as observed in Table 2.
Finally, in the HD[(As; Te; Sb; Sn)- X] ER-LD[(Ga; Mg; In; Cd)- X] BR and for physical conditions given in Equations (45.1, 45.2), our numerical results of $n_I$, $J_{scI}$, $F_I$, and $\eta_I$, are computed, using Equations (42, 41, 43, 44), respectively, and reported in the following Tables 3, 4, 5, and 6.

Tables 3, 4, 5, 6 are in the Appendix 1.

In summary, for a given (X≡ CdTe, or CdSe)- crystal at $T=300$ K, one notes that, with an increasing $r_a$(d):

- $J_{oI}$($oII$) decreases, due to the increase in the IBG (as observed in Tables 1 and 2),

- and then, as observed in Tables 3, 4, 5, and 6, for a given $V_{oc}$, the photovoltaic conversion factor (PVCF), $n_{I(II)}$, decreases, giving rise to the increase in $\eta_{I(II)}$, in good agreement with an important result, obtained by Shockley and Queisser (1961)], stating that, for $IBG<1.6$ eV, $\eta_{I(II)}$ increases with increasing IBG.

Finally, from the well-known Carnot’s theorem, being obtained from the second principle of the thermodynamics, the maximum efficiency of a heat engine operating between hot (H) and cold (C) reservoirs is the ratio of the temperature difference between the reservoirs to the H-reservoir temperature, $T_H$, expressed as:

$$\eta_{\text{max}} = \frac{T_H-T_C}{T_H}$$

suggesting that $\eta_{\text{max}}$ and $T_H$ have the same variations.

Here, for example, from Tables 4 and 6, and for $a= b=0$, chosen in Eq. (43) for the fill factor $F$, the values of $\eta_{\text{max}}$ are found to be given by: 26.55 % and 23.69 %, obtained at $V_{oc}=0.82$ V and 0.89 V, respectively, and at $T=300$ K $\equiv T_C$. Therefore, from Eq. (46), with $\eta_{\text{max}} = 26.55$ % and 23.69 %, one obtains: $T_H=408.4$ K and 393.1 K, respectively. Thus, as noted above, $\eta_{\text{max}}$ and $T_H$ both increase with an increasing IBG, for each (X≡ CdTe, or CdSe)- crystal at $T=300$ K $\equiv T_C$.

References


Van Cong, H. (1991). New series representation of Fermi-Dirac integral $F_j(\alpha)$ for arbitrary $|\alpha| > 1$, and its effect on $F_j(\alpha \geq 0)$ for integer $|\alpha| \geq 0$. *Solid-State Electron.*, 34, 489-492.


Table 3. In the HD [(Ga; Mg; In; Cd)-CdTe] ER-LD [(As; Te; Sb; Sn)-CdTe] BR and for physical conditions given in Eq. (45.3), our numerical results of \( n_{II} \), \( J_{scII} \), \( F_{II} \), and \( \eta_{II} \), are computed, using Equations (42, 41, 43, 44), respectively. Here, it is noted that, for a given \( V_{oc} \), and with increasing \( r_{a(d)} \), the function \( n_{II} \) decreases, while other functions \( J_{scII} \), \( F_{II} \), and \( \eta_{II} \) increase, being due to the \( r_{a(d)} \) effect, suggesting thus the new obtained results.

<table>
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<tr>
<th>( V_{oc}(V) )</th>
<th>( n_{II} )</th>
<th>( J_{scII}(mA/cm^2) )</th>
<th>( F_{II} (%) )</th>
<th>( \eta_{II} (%) )</th>
</tr>
</thead>
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<td>0.73</td>
<td>0.714; 0.703; 0.702; 0.701</td>
<td>21.60; 21.60; 21.60; 21.60</td>
<td>88.42; 88.55; 88.57; 88.57</td>
<td>13.94; 13.96; 13.97; 13.97</td>
</tr>
<tr>
<td>0.81</td>
<td>0.782; 0.770; 0.769; 0.769</td>
<td>35.62; 35.88; 35.91; 35.92</td>
<td>88.53; 88.66; 88.68; 88.68</td>
<td>25.54; 25.76; 25.79; 25.80</td>
</tr>
<tr>
<td>0.82</td>
<td>0.792; 0.780; 0.779; 0.778</td>
<td>35.33; 35.58; 35.61; 35.62</td>
<td>88.53; 88.66; 88.68; 88.68</td>
<td>25.65; 25.87; 25.90; 25.90</td>
</tr>
<tr>
<td>0.83</td>
<td>0.802; 0.790; 0.789; 0.788</td>
<td>34.78; 35.02; 35.04; 35.05</td>
<td>88.53; 88.66; 88.67; 88.68</td>
<td>25.56; 25.77; 25.79; 25.80</td>
</tr>
<tr>
<td>0.8759</td>
<td>0.849; 0.837; 0.835; 0.835</td>
<td>30.10; 30.23; 30.24; 30.25</td>
<td>88.49; 88.63; 88.64; 88.65</td>
<td>23.33; 23.46; 23.48; 23.49</td>
</tr>
<tr>
<td>1.2</td>
<td>1.228; 1.210; 1.208; 1.207</td>
<td>3.725; 3.619; 3.607; 3.602</td>
<td>88.01; 88.15; 88.16; 88.17</td>
<td>3.934; 3.828; 3.816; 3.811</td>
</tr>
</tbody>
</table>

In Eq. (43), obtained for the \( a=1 \) and \( b=0.72 \), according to an ideal solar cells (Green, 1981).

<table>
<thead>
<tr>
<th>( V_{oc}(V) )</th>
<th>( n_{II} )</th>
<th>( J_{scII}(mA/cm^2) )</th>
<th>( F_{II} (%) )</th>
<th>( \eta_{II} (%) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.73</td>
<td>0.714; 0.703; 0.702; 0.701</td>
<td>21.60; 21.60; 21.60; 21.60</td>
<td>90.70; 90.80; 90.81; 90.82</td>
<td>14.30; 14.32; 14.32; 14.32</td>
</tr>
<tr>
<td>0.81</td>
<td>0.782; 0.770; 0.769; 0.769</td>
<td>35.62; 35.88; 35.91; 35.92</td>
<td>90.79; 90.89; 90.90; 90.90</td>
<td>26.19; 26.41; 26.44; 26.45</td>
</tr>
<tr>
<td>0.82</td>
<td>0.792; 0.780; 0.779; 0.778</td>
<td>35.33; 35.58; 35.61; 35.62</td>
<td>90.78; 90.89; 90.90; 90.90</td>
<td>26.30; 26.51; 26.54; 26.55</td>
</tr>
<tr>
<td>0.83</td>
<td>0.802; 0.790; 0.789; 0.788</td>
<td>34.78; 35.02; 35.04; 35.05</td>
<td>90.78; 90.88; 90.89; 90.90</td>
<td>26.21; 26.41; 26.44; 26.45</td>
</tr>
<tr>
<td>0.8759</td>
<td>0.849; 0.837; 0.835; 0.835</td>
<td>30.10; 30.23; 30.24; 30.25</td>
<td>90.76; 90.86; 90.87; 90.87</td>
<td>23.93; 24.06; 24.07; 24.08</td>
</tr>
<tr>
<td>1.2</td>
<td>1.228; 1.210; 1.208; 1.207</td>
<td>3.725; 3.619; 3.607; 3.602</td>
<td>90.39; 90.49; 90.50; 90.51</td>
<td>4.040; 3.930; 3.917; 3.912</td>
</tr>
</tbody>
</table>

In Eq. (43), obtained for \( F = a=b=0 \), according to highest values of \( F \).
Table 4. In the HD [(As; Te; Sb; Sn)-CdTe] ER-LD [(Ga; Mg; In; Cd)-CdTe] BR and for physical conditions given in Eq. (45.3), our numerical results of $n_{III}$, $I_{scII}$, $F_{II}$, and $\eta_{III}$, are computed, using Equations (42, 41, 43, 44), respectively. Here, on notes that, for a given $V_{oc}$ and with increasing $r_{a(d)}$, the function $n_{III}$ decreases, while other functions $I_{scII}$, $F_{II}$, and $\eta_{III}$ increase, being due to the $r_{d(a)} -$ effect, suggesting thus the new obtained results.

<table>
<thead>
<tr>
<th>$V_{oc}(V)$</th>
<th>$n_{I}$</th>
<th>$I_{sc}(mA/cm^2)$</th>
<th>$F_{I}(%)$</th>
<th>$\eta(%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.87</td>
<td>0.673; 0.667; 0.666; 0.666</td>
<td>24.32; 24.32; 24.32; 24.32</td>
<td>90.34; 90.41; 90.42; 90.42</td>
<td>19.11; 19.13; 19.13; 19.13</td>
</tr>
<tr>
<td>0.88</td>
<td>0.679; 0.673; 0.672; 0.672</td>
<td>27.21; 27.24; 27.24; 27.24</td>
<td>90.36; 90.43; 90.44; 90.44</td>
<td>21.64; 21.68; 21.68; 21.68</td>
</tr>
<tr>
<td>0.89</td>
<td>0.686; 0.680; 0.679; 0.679</td>
<td>28.70; 28.74; 28.75; 28.75</td>
<td>90.37; 90.44; 90.44; 90.45</td>
<td><strong>23.08; 23.13; 23.14; 23.14</strong></td>
</tr>
<tr>
<td>1.03</td>
<td>0.799; 0.792; 0.791; 0.791</td>
<td>20.17; 20.14; 20.14; 20.14</td>
<td>90.32; 90.38; 90.39; 90.39</td>
<td>18.76; 18.75; 18.75; 18.75</td>
</tr>
<tr>
<td>1.1</td>
<td>0.861; 0.853; 0.852; 0.852</td>
<td>13.45; 13.38; 13.37; 13.37</td>
<td>90.25; 90.32; 90.33; 90.33</td>
<td>13.35; 13.29; 13.29; 13.29</td>
</tr>
<tr>
<td>1.2</td>
<td>0.951; 0.942; 0.941; 0.941</td>
<td>7.136; 7.061; 7.052; 7.049</td>
<td>90.16; 90.22; 90.23; 90.24</td>
<td>7.720; 7.635; 7.636; 7.633</td>
</tr>
</tbody>
</table>

In Eq. (43), obtained for the F as a=1 and b=0.72, according to an ideal solar cells (Green, 1981).

<table>
<thead>
<tr>
<th>$n_{I}$</th>
<th>$I_{sc}(mA/cm^2)$</th>
<th>$F_{I}(%)$</th>
<th>$\eta(%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.87</td>
<td>0.673; 0.667; 0.666; 0.666</td>
<td>24.32; 24.32; 24.32; 24.32</td>
<td>90.34; 90.41; 90.42; 90.42</td>
</tr>
<tr>
<td>0.88</td>
<td>0.679; 0.673; 0.672; 0.672</td>
<td>27.21; 27.24; 27.24; 27.24</td>
<td>90.36; 90.43; 90.44; 90.44</td>
</tr>
<tr>
<td>0.89</td>
<td>0.686; 0.680; 0.679; 0.679</td>
<td>28.70; 28.74; 28.75; 28.75</td>
<td>90.37; 90.44; 90.44; 90.45</td>
</tr>
<tr>
<td>1.03</td>
<td>0.799; 0.792; 0.791; 0.791</td>
<td>20.17; 20.14; 20.14; 20.14</td>
<td>90.32; 90.38; 90.39; 90.39</td>
</tr>
<tr>
<td>1.1</td>
<td>0.861; 0.853; 0.852; 0.852</td>
<td>13.45; 13.38; 13.37; 13.37</td>
<td>90.25; 90.32; 90.33; 90.33</td>
</tr>
<tr>
<td>1.2</td>
<td>0.951; 0.942; 0.941; 0.941</td>
<td>7.136; 7.061; 7.052; 7.049</td>
<td>90.16; 90.22; 90.23; 90.24</td>
</tr>
</tbody>
</table>

In Eq. (43), obtained for F as a=b=0, according to highest values of F.

<table>
<thead>
<tr>
<th>$n_{I}$</th>
<th>$I_{sc}(mA/cm^2)$</th>
<th>$F_{I}(%)$</th>
<th>$\eta(%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.87</td>
<td>0.673; 0.667; 0.666; 0.666</td>
<td>24.32; 24.32; 24.32; 24.32</td>
<td>92.18; 92.23; 92.24; 92.24</td>
</tr>
<tr>
<td>0.88</td>
<td>0.679; 0.673; 0.672; 0.672</td>
<td>27.21; 27.24; 27.24; 27.24</td>
<td>92.19; 92.24; 92.25; 92.25</td>
</tr>
<tr>
<td>0.89</td>
<td>0.686; 0.680; 0.679; 0.679</td>
<td>28.70; 28.74; 28.75; 28.75</td>
<td>92.20; 92.25; 92.26; 92.26</td>
</tr>
<tr>
<td>1.03</td>
<td>0.799; 0.792; 0.791; 0.791</td>
<td>20.17; 20.14; 20.14; 20.14</td>
<td>92.16; 92.21; 92.22; 92.22</td>
</tr>
<tr>
<td>1.1</td>
<td>0.861; 0.853; 0.852; 0.852</td>
<td>13.45; 13.38; 13.37; 13.37</td>
<td>92.11; 92.16; 92.17; 92.17</td>
</tr>
<tr>
<td>1.2</td>
<td>0.951; 0.942; 0.941; 0.941</td>
<td>7.136; 7.061; 7.052; 7.049</td>
<td>92.03; 92.09; 92.09; 92.09</td>
</tr>
</tbody>
</table>
Table 5. In the HD[(Sn; Te; Sb; Sn)- CdSe] ER-LD[(Ga; Mg; In; Cd)- CdSe] BR and for physical conditions given in Eq. (45.3), our numerical results of $n_I$, $J_{sc}$, $F_I$, and $\eta_I$, are computed, using Equations (42, 41, 43, 44), respectively. Here, on notes that, for a given $V_{oc}$ and with increasing $r_d(a)$, the function $n_I$ decreases, while other functions $J_{sc}$, $F_I$, and $\eta_I$ increase, being due to the $r_d(a)$-effect, suggesting thus the new obtained results

<table>
<thead>
<tr>
<th>$V_{oc}$(V)</th>
<th>$n_I$</th>
<th>$J_{sc}$(mA/cm²)</th>
<th>$F_I$(%)</th>
<th>$\eta_I$(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>In Eq. (43), obtained for the F as $a=1$ and $b=0.72$, according to an ideal solar cells (Green, 1981).</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.87</td>
<td>n+p</td>
<td>Se$^+$Ga; Te$^+$Mg; Sb$^+$In; Sn$^+$Cd</td>
<td>0.673; 0.667; 0.666</td>
<td>24.32; 24.32; 24.32; 24.32</td>
</tr>
<tr>
<td>0.88</td>
<td>n+p</td>
<td>Se$^+$Ga; Te$^+$Mg; Sb$^+$In; Sn$^+$Cd</td>
<td>0.679; 0.673; 0.672</td>
<td>27.21; 27.24; 27.24; 27.24</td>
</tr>
<tr>
<td>0.89</td>
<td>n+p</td>
<td>0.686; 0.680; 0.679; 0.679</td>
<td>28.70; 28.74; 28.75; 28.75</td>
<td>90.37; 90.44; 90.44; 90.45</td>
</tr>
<tr>
<td>1.03</td>
<td>n+p</td>
<td>0.799; 0.792; 0.791; 0.791</td>
<td>20.17; 20.14; 20.14; 20.14</td>
<td>90.32; 90.38; 90.39; 90.39</td>
</tr>
<tr>
<td>1.1</td>
<td>n+p</td>
<td>0.861; 0.855; 0.852; 0.852</td>
<td>13.45; 13.38; 13.37; 13.37</td>
<td>90.25; 90.32; 90.33; 90.33</td>
</tr>
<tr>
<td>1.2</td>
<td>n+p</td>
<td>0.951; 0.942; 0.941; 0.941</td>
<td>7.136; 7.061; 7.052; 7.049</td>
<td>90.16; 90.22; 90.23; 90.24</td>
</tr>
<tr>
<td>In Eq. (43), obtained for F as $a=b=0$, according to highest values of F.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.87</td>
<td>n+p</td>
<td>0.673; 0.667; 0.666</td>
<td>24.32; 24.32; 24.32; 24.32</td>
<td>90.18; 90.23; 90.24; 90.24</td>
</tr>
<tr>
<td>0.88</td>
<td>n+p</td>
<td>0.679; 0.673; 0.672</td>
<td>27.21; 27.24; 27.24; 27.24</td>
<td>90.19; 90.24; 90.25; 90.25</td>
</tr>
<tr>
<td>0.89</td>
<td>n+p</td>
<td>0.686; 0.680; 0.679; 0.679</td>
<td>28.70; 28.74; 28.75; 28.75</td>
<td>90.20; 90.25; 90.26; 90.26</td>
</tr>
<tr>
<td>1.03</td>
<td>n+p</td>
<td>0.799; 0.792; 0.791; 0.791</td>
<td>20.17; 20.14; 20.14; 20.14</td>
<td>90.16; 90.21; 90.22; 90.22</td>
</tr>
<tr>
<td>1.1</td>
<td>n+p</td>
<td>0.861; 0.855; 0.852; 0.852</td>
<td>13.45; 13.38; 13.37; 13.37</td>
<td>90.11; 90.16; 90.17; 90.17</td>
</tr>
<tr>
<td>1.2</td>
<td>n+p</td>
<td>0.951; 0.942; 0.941; 0.941</td>
<td>7.136; 7.061; 7.052; 7.049</td>
<td>90.03; 90.09; 90.09; 90.09</td>
</tr>
</tbody>
</table>
Table 6. In the HD[(Ga; Mg; In; Cd)-CdSe] ER-LD[(Se; Te; Sb; Sn)-CdSe] BR and for physical conditions given in Eq. (45.3), our numerical results of $n_{III}$, $J_{sc(III)}$, $F_{II}$, and $\eta_{III}$, are computed, using Equations (42, 41, 43, 44), respectively. Here, on notes that, for a given $V_{oc}$ and with increasing $r_{a(d)}$, the function $n_{III}$ decreases, while other functions $J_{sc(III)}$, $F_{II}$, and $\eta_{III}$ increase, being due to the $r_{a(d)}$-effect, suggesting thus the new obtained results.

<table>
<thead>
<tr>
<th>$V_{oc}$ (V)</th>
<th>$n_{III}$</th>
<th>$J_{sc(III)}$ (mA/cm$^2$)</th>
<th>$F_{II}$ (%)</th>
<th>$F_{III}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>In Eq. (43), obtained for the F as $a=1$ and $b=0.72$, according to an ideal solar cells (Green, 1981).</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p+n</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.87</td>
<td>0.665; 0.661; 0.660; 0.659</td>
<td>24.32; 24.32; 24.32; 24.32</td>
<td>90.43; 90.48; 90.49; 90.50</td>
<td>19.13; 19.14; 19.14; 19.15</td>
</tr>
<tr>
<td>0.88</td>
<td>0.671; 0.667; 0.666; 0.665</td>
<td>27.26; 27.28; 27.29; 27.29</td>
<td>90.45; 90.49; 90.50; 90.51</td>
<td>21.70; 21.73; 21.73; 21.74</td>
</tr>
<tr>
<td>0.89</td>
<td>0.678; 0.674; 0.673; 0.672</td>
<td>28.79; 28.82; 28.82; 28.83</td>
<td>90.46; 90.50; 90.51; 90.52</td>
<td>23.17; 23.21; 23.22; 23.23</td>
</tr>
<tr>
<td>1.03</td>
<td>0.790; 0.785; 0.784; 0.783</td>
<td>20.22; 20.20; 20.19; 20.20</td>
<td>90.40; 90.45; 90.46; 90.47</td>
<td>18.82; 18.82; 18.81; 18.81</td>
</tr>
<tr>
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<td>90.34; 90.39; 90.40; 90.41</td>
<td>13.34; 13.30; 13.29; 13.28</td>
</tr>
<tr>
<td>1.2</td>
<td>0.940; 0.934; 0.933; 0.932</td>
<td>7.069; 7.017; 7.000; 6.995</td>
<td>90.24; 90.29; 90.30; 90.31</td>
<td>7.655; 7.603; 7.591; 7.581</td>
</tr>
<tr>
<td>In Eq. (43), obtained for F as $a=b=0$, according to highest values of F.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p+n</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.87</td>
<td>0.665; 0.661; 0.660; 0.659</td>
<td>24.32; 24.32; 24.32; 24.32</td>
<td>92.23; 92.28; 92.29; 92.30</td>
<td>19.52; 19.52; 19.53; 19.53</td>
</tr>
<tr>
<td>0.88</td>
<td>0.671; 0.667; 0.666; 0.665</td>
<td>27.26; 27.28; 27.29; 27.29</td>
<td>92.26; 92.29; 92.30; 92.31</td>
<td>22.13; 22.16; 22.16; 22.17</td>
</tr>
<tr>
<td>0.89</td>
<td>0.678; 0.674; 0.673; 0.672</td>
<td>28.79; 28.82; 28.82; 28.83</td>
<td>92.26; 92.30; 92.31; 92.32</td>
<td>23.64; 23.67; 23.68; 23.69</td>
</tr>
<tr>
<td>1.03</td>
<td>0.790; 0.785; 0.784; 0.783</td>
<td>20.22; 20.20; 20.19; 20.20</td>
<td>92.22; 92.26; 92.27; 92.28</td>
<td>19.20; 19.19; 19.19; 19.19</td>
</tr>
<tr>
<td>1.1</td>
<td>0.851; 0.845; 0.844; 0.843</td>
<td>13.42; 13.37; 13.36; 13.35</td>
<td>92.18; 92.21; 92.22; 92.23</td>
<td>13.61; 13.57; 13.56; 13.55</td>
</tr>
<tr>
<td>1.2</td>
<td>0.940; 0.934; 0.933; 0.932</td>
<td>7.069; 7.017; 7.000; 6.995</td>
<td>92.10; 92.14; 92.15; 92.15</td>
<td>7.813; 7.759; 7.746; 7.736</td>
</tr>
</tbody>
</table>