Possibilistic Aggregate Production Planning Considering Dynamic Workforce with Fuzzy Demand

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Abstract:
This Aggregate Production Planning (APP) is a combination of Possibilistic Linear Programming (PLP), Fuzzy Goal Programming (FGP), and the Perfume Accounting System (PAS) to maximize profit. APP involves strategic decisions on Production levels, Inventory management, and Resource allocation to meet client demand while maximizing profit. Traditional planning models face significant challenges due to the uncertainties and complexities inherent in real world production environments. In this paper, there is an integration of PLP, Fuzzy Goal Programming, and throughput accounting system to overcome these challenges. At the very end of the paper, based on the data received from the company, the derived findings were by using Lingo Version 18 software. The model includes possibility distributions of the input parameters. Decision-makers can take into account the uncertainty and imprecision in demand forecasts and dynamic workforce in maximizing profit while taking into account risk tolerance.

Keywords: Aggregate production planning, fuzzy demands, capacity utilization, Decision maker, throughput accounting

Introduction
Aggregate Production Planning (APP) is a crucial component of manufacturing and production management that entails choosing the best production levels, labor distribution, and inventory management to satisfy the uncertain demand while minimizing costs and maximizing efficiency. Fuzzy logic must be incorporated into the APP process when dealing with concerns about a dynamic workforce and unpredictable, imprecise demand. Adjusting the number of employees, their abilities, and shifts in response to shifting demand levels is known as dynamic workforce management. Traditional methods frequently take into account a static workforce, which results in ineffective resource use and potential labor shortages during peak demand. By taking into account workforce dynamics, businesses may effectively match labor resources with changing production demands.

Review of Related Works
Aggregate production planning (APP) is a medium-term production decision in a manufacturing organization that establishes the production rate, inventory level, amount of subcontractors, and workforce level in a particular time according to a number of constraints. "Aggregate" refers to the preparation done for two or more manufacturing categories. Determining output levels across all categories to meet current, specific demands is the goal of aggregate production planning. APP governs the best way to meet forecast demand in the intermediate future, often from 6 to 24 months ahead, by
adjusting regular and overtime production rates, inventory levels, labor levels, subcontracting and backordering rates, and other controllable variables (Wang et al., 2005). The primary inputs of APP are market demands and the manufacturing plan to meet those expectations. (Leung et al., 2003). The traditional models for APP often assume deterministic demand and a fixed workforce, which might not be realistic in today's volatile market conditions.

For the purpose of production-distribution planning, Du, M., and Leung, S. Y. S. (2007) provide a fuzzy multi-objective model that takes into account workforce dynamics, unpredictable demand, and production capacity. The goal of the concept is to balance consumer happiness and production costs as efficiently as possible. In their study on the make-to-order sector, Yen, B. P. C., and Liu (2009) provide a fuzzy APP model that takes into account both imprecise demand and dynamic workforce modifications. The study shows how well fuzzy logic captures demand uncertainty and its influence on production choices. Fuzzy multi-objective linear programming approaches are introduced for issues involving aggregate production planning by Taleizadeh, A. A., and Pentico, D. W. (2012). In order to help decision-makers make sound and reliable planning decisions, the research tackles labor dynamics, production capacity, and ambiguous demand. A fuzzy multi-objective model that incorporates workforce allocation choices into the process of aggregate production planning is presented in a study by Guneri, A. F., and Olgun, H. (2018). The research optimizes for both profit and customer pleasure while taking a variety of elements, such as labor prices, production rates, and ambiguous demand, into consideration.

Considering dynamic labor management with fuzzy demand modeling makes the already difficult process of aggregate production planning considerably more difficult. The literature study emphasizes the significance of including these components in planning models to improve alignment between production capabilities and changing consumer needs. In the context of planning collective production, fuzzy logic offers a useful tool for dealing with uncertain data and imprecise information. This enhances the reliability and effectiveness of decisions. For the planning process to fully address the dynamic nature of both labor and demand issues, further research is required. This work investigates the optimal profit level, using proper account system in the APP system under an imprecise demand and a dynamic workforce scenario using Possibilistic Programming model.

Method and Procedure
The Financial Aspects of the Theory of Constraints

The amount of money spent on a system to increase its capacity is known as investment, and throughput accounting lays a lot of attention on it. The following formulae are used by throughput accounting in conjunction with throughput, entirely variable costs, and operational expenditures for a variety of accounting decisions:

- **Throughput (TP)** = **Revenue (R)** − **Totally Variable Expenses (TVE)**
- **Net Profit (NP)** = **Throughput (TP)** − **Operating Expenses (OE)**
- **Return on Investment (RoI)** = **Net Profit / Investment**
- **Productivity (Pr)** = **Throughput / Operational Expense**
- **Inventory Turns (IT)** = **Throughput / Inventory Value**

Goldratt's strategy demanded a shift in emphasis in order to redefine accounting principles. The goal for managers should be to increase throughput while lowering operating costs and inventories. The latter two, however, must be maintained at a certain minimum level to prevent a decrease in throughput, thus there is very little room for reduction in those two areas. These might be viewed as restrictions on the suggested model. The weakest link in the system chain is a constraint (Dettemer, 1997). There are three different kinds of constraints: material, resource, and policy (paradigm) (Woeppel, 2001).
Moreover, financial limitations are crucial for real-world issues (Fung, et al., 2003).

**Assumptions and Problem Definition**

Following the findings of a real-world case study, the following presumptions are made for the mathematical model of the suggested APP problem.

- Production planning is done in a time horizon of $T$ time periods ($\forall t = 1, 2, ..., T$).
- There is a Batch production system capable of producing all kinds of $N$ types of products.
- Market demand can be fulfilled or backordered, however no backorder in the last $t$ is allowed.
- There are two working shifts; Regular time production and Over time production.
- A warehouse is allowed for holding final products.
- In advance, the holding cost of inventories are determined and well known.
- The workforce accommodates various skill levels ($k \text{ – levels}$).
- Workers salary is independent of unit production cost.
- At each period $T$, Production quantity is considered more of the safety stock for finished products.
- Hiring and firing of Manpower based on product demand is eligible and there is an allowable limit.
- In each period $T$, the shortage of production is recovered by overtime production in each shift.
- In each period $T$, the nominal and actual capacity of production machines is not the same due to unforeseen failures.
- If an unforeseen failure occurs during a shift the repair process is completed in the next. This may stop, reduce, or decrease the production rate during maintenance actions.

- The impreciseness and uncertainty of real-world problem and confliction of different objectives are modeled using fuzzy goals.
- Linear membership functions are defined for fuzzy goals.

**Parameters, Indices, Decision Variables and Notations**

### Table 1. Set of indices

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>Number of periods in the planning horizon; $t = 1, 2, ..., T$</td>
</tr>
<tr>
<td>$i$</td>
<td>Number of product types; $i = 1, 2, ..., I$</td>
</tr>
<tr>
<td>$m$</td>
<td>Raw material type; $m = 1, 2, ..., M$</td>
</tr>
<tr>
<td>$q$</td>
<td>Types of shifts; $q \in {1, 2}$</td>
</tr>
<tr>
<td>$w$</td>
<td>Types of warehouse; $w = 1, 2, ..., W$</td>
</tr>
<tr>
<td>$k$</td>
<td>Skill levels of workers; $k = 1, 2, ..., K$</td>
</tr>
<tr>
<td>$j$</td>
<td>Number of objective Functions; $j = 1, 2, 3$</td>
</tr>
</tbody>
</table>

### Table 2. Notation for parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$PW_i$</td>
<td>Fraction of the Product $i$ wasted during production in period $t$</td>
</tr>
<tr>
<td>$WI_i$</td>
<td>Fraction of the Product $i$ wasting in inventory in period $t$</td>
</tr>
<tr>
<td>$CoO_{iq}$</td>
<td>Cost of Overhead(fixed) Production; for product $i$ in shift $q$</td>
</tr>
<tr>
<td>$DoP_{it}$</td>
<td>Demand of product $i$ in period $t$</td>
</tr>
<tr>
<td>$CoB_{it}$</td>
<td>Cost of Backordering; for product $i$ in period $t$</td>
</tr>
<tr>
<td>$SRe_i$</td>
<td>Sales Revenue for product $i$ (₦/unit)</td>
</tr>
<tr>
<td>$E_t$</td>
<td>cumulative investment in tools and equipment in period $t$ (currency unit)</td>
</tr>
<tr>
<td>$PrT_i$</td>
<td>Process time of product $i$ in period $t$</td>
</tr>
<tr>
<td>$BUL_t$</td>
<td>The Budget upper limit in period $t$</td>
</tr>
<tr>
<td>$AsP_{it}$</td>
<td>Allowable shortage of product $i$ in period $t$</td>
</tr>
<tr>
<td>$AMW_{it}$</td>
<td>Available Maximum workforce in period $t$</td>
</tr>
<tr>
<td>$AMW_{it}$</td>
<td>Available Minimum workforce in period $t$</td>
</tr>
<tr>
<td>$WaO$</td>
<td>workforce that are available for overtime (in percentage)</td>
</tr>
<tr>
<td>$CoW_{kt}$</td>
<td>Cost of workforce of level $k$ in period $t$</td>
</tr>
<tr>
<td>$CoH_{kt}$</td>
<td>Cost of Hiring workforce of level $k$ in period $t$</td>
</tr>
<tr>
<td>$CoF_{kt}$</td>
<td>Cost of firing workforce of level $k$ in period $t$</td>
</tr>
<tr>
<td>$CoM_{mwt}$</td>
<td>Cost for raw material type $m$ in period $t$ in warehouse $w$</td>
</tr>
</tbody>
</table>
Table 3. Decision variable Notation

<table>
<thead>
<tr>
<th>Decision variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_{iqt}$</td>
<td>Number of product $i$ produced in shift $q$ of period $t$</td>
</tr>
<tr>
<td>$XB_{iqt}$</td>
<td>Number batches of product $i$ produced in shift $q$ of period $t$</td>
</tr>
<tr>
<td>$B_{it}$</td>
<td>Backorder level of product $i$ in period $t$</td>
</tr>
<tr>
<td>$XW_{kt}$</td>
<td>Number of available workers of level $k$ in period $t$</td>
</tr>
<tr>
<td>$XH_{kt}$</td>
<td>Number of hired workers of level $k$ in period $t$</td>
</tr>
<tr>
<td>$XF_{kt}$</td>
<td>Number of fired workers of level $k$ in period $t$</td>
</tr>
<tr>
<td>$XR_{mtw}$</td>
<td>Inventory level of raw material type $m$ at the end of period $t$ in warehouse $w$</td>
</tr>
<tr>
<td>$XP_{itw}$</td>
<td>Inventory level of finished-product $i$ in period $t$ in warehouse $w$</td>
</tr>
</tbody>
</table>

Model Formulation

Total income less the cost of the materials you purchased is your throughput (TP). Equation (1) can be used to numerically express the throughput (TP).

\[
TP = \sum_{t=1}^{T} \sum_{i=1}^{I} SRe_{i}DoP_{it} - \sum_{t=1}^{T} \sum_{i=1}^{I} SRe_{i}B_{it} - \sum_{m=1}^{M} \sum_{w=1}^{W} \sum_{t=1}^{T} CoM_{mwt}X_{iqt}
\]  

(1)

The first two terms reflect total sales revenue based on total demands and lost sales at the conclusion of the planned horizon. The last term denotes the cost of materials, which includes the cost of materials needed for both regular and overtime production.

The definition of inventory is "the entire money the system invests," which includes the money spent on all assets (such as structures, machinery, and fixtures) as well as on raw materials and parts (Woeppel, 2001). TOC inventory differs from traditional inventory in that it encompasses all assets in addition to raw materials, work-in-progress, and finished goods.

\[
IN = \frac{1}{T} \left[ \sum_{m=1}^{M} \sum_{w=1}^{W} \sum_{t=1}^{T} CoR_{mwt}XR_{mwt} + \sum_{t=1}^{T} E_{t} \right]
\]  

(2)
The APP can take assets like small machines, material handling equipment and other instruments into account. Strategic or long-range planning takes into account structures and huge machineries.

The capacity of a machine or system can be increased and a bottleneck reduced by investing in tools and equipment. Equation (2) represents the typical inventory investment (IN) in terms of TOC. Whereas the second term indicates the investment in tools and equipment, the first term represents the investment in raw materials.

$$OE = \sum_{k=1}^{K} \sum_{t=1}^{T} CoW_{kt}XW_{kt} + \sum_{k=1}^{K} \sum_{t=1}^{T} CoH_{kt}XH_{kt} + \sum_{k=1}^{K} \sum_{t=1}^{T} CoF_{kt}XF_{kt} + \sum_{l=1}^{I} \sum_{w=1}^{W} \sum_{t=1}^{T} CoP_{lw}XP_{lw}$$

The entire amount of money needed to convert inventory into throughput is referred to as operating expenses. All direct and indirect payroll expenses, purchases, overhead and time related expenses are involved. Equation (3) represents operating expense (OE). It covers all labour, overtime, holding expenses for inventory, backordering and fixed overhead costs.

Aggregate Production Planning Considering Throughput Accounting

Often, the objective function of APP problems is chosen to be the revenue, cost, or profit function. The profit function is the most desirable of these objective functions (Phruksaphanrat et al., 2006). Thus, the objective function of the suggested APP model is the Net Profit (NP). Throughput (TP) minus Operational expenses (OE) equals Net Profit (NP). The profit function includes two TOC metrics. Inventory is the final factor, which should also be taken into account. According to TOC, inventory refers to all financial investments made by the system, including those made in tools and equipment. It is incorporated into the model as constraints.

Maximize Net Profit (NP)

$$Z = \sum_{l=1}^{I} \sum_{t=1}^{T} SR_{le}Do_{lt} - \sum_{l=1}^{I} \sum_{t=1}^{T} SR_{le}B_{lt} - \sum_{m=1}^{M} \sum_{w=1}^{W} \sum_{t=1}^{T} CoM_{mt}X_{mt} - \left[ \sum_{k=1}^{K} \sum_{t=1}^{T} CoW_{kt}XW_{kt} + \sum_{k=1}^{K} \sum_{t=1}^{T} CoH_{kt}XH_{kt} + \sum_{k=1}^{K} \sum_{t=1}^{T} CoF_{kt}XF_{kt} + \sum_{l=1}^{I} \sum_{w=1}^{W} \sum_{t=1}^{T} CoP_{lw}XP_{lw} + \sum_{m=1}^{M} \sum_{w=1}^{W} \sum_{t=1}^{T} CoR_{mt}XR_{mt} + \sum_{i=1}^{I} \sum_{t=1}^{T} CoB_{it}B_{it} + \sum_{i=1}^{I} \sum_{q\in\{1,2\}} \sum_{t=1}^{T} CoO_{iq}X_{iq} \right]$$

Constraints

The Labor-force Constraints are considered as follows:

$$\sum_{k=1}^{K} XW_{kt} \leq AMW_{t}, \quad \forall t$$
\[
\sum_{k=1}^{K} XW_{kt} \geq AMW_t, \quad \forall t \tag{6}
\]

\[
XW_{kt} = XW_{k(t-1)} + XH_{kt} - XF_{kt}, \quad \forall k, \forall t, t > 1 \tag{7}
\]

\[
XW_{kt} - XW_{k(t-1)} \leq FoW_t \times XW_{kt}, \quad \forall k, \forall t, t > 1 \tag{8}
\]

Constraints (5) attest that the total labor utilized during period \(t\) does not exceed the total workforce that is available. In a similar vein, (6) guarantees that in period \(t\), the employed workforce exceeds the available minimum workforce. Set of Constraints (7) is a workforce level balance equation that assures that the workforce with skill level \(k\) available during a given period is equal to the workforce with the same skill level \(k\) during the previous period plus the change in workforce level during the current period. The change in workforce level in each planning period cannot be greater than a benchmark number of workers in the present period, according to constraint number seven.

**Time Constraints**

\[
\sum_{i=1}^{I} PrT_{it} \times X_{iqt} \leq \sum_{k=1}^{K} ArT_{qt} \times XW_{kt}, \quad \forall t, q = 1 \tag{9}
\]

\[
\sum_{i=1}^{I} PrT_{it} \times X_{iqt} \leq \sum_{k=1}^{K} ArT_{qt} \times WaO \times XW_{kt}, \quad \forall t, q = 2 \tag{10}
\]

The relationships mentioned above make sure that each working shift's necessary production time is less than or equal to the available regular production time and overtime.

**Inventory Constraints**

\[
XP_{iwt} = XP_{iwl(t-1)} + \sum_{q=1,2} X_{iqt} - B_{it} - DoP_{it}, \quad \forall i, \forall w, \quad t > 1 \tag{11}
\]

\[
XR_{mwt} = XR_{mw(t-1)} + \sum_{q=1,2} X_{iq(t-1)} - uRM_{im}, \quad \forall i, \forall w, \quad t > 1 \tag{12}
\]

\[
SSR_m \leq \sum_{w \in W} XR_{mwt}, \quad \forall m, \forall t, \tag{13}
\]

Constraints (11) ensures that the amount of finished product type \(I\) in period \(t\) in warehouse \(w\) is equal to the amount of finished product type \(I\) in period \(t - 1\) in warehouse \(w\) plus the quantity of produced finished goods type \(I\) in period \(t\) in both working shifts, less the amount of product type \(I\) in period \(t\) that is on backorder and the quantity of produced finished goods type \(I\) in period \(t\) in both working shifts. A set of limitations (12) assures that there is a balance between raw materials, and (13) guarantees that the safety stock of raw materials in warehouses is satisfied.
Production Constraint

\[ SSP_i \leq \sum_{q \in [1,2]} X_{iqt}, \quad \forall i, \forall t, \]  

\[ DoP_{it} \leq \left(1 - \frac{DrFi}{\beta_i}\right) \sum_{q \in [1,2]} X_{iqt} + XP_{i(t-1)}, \quad \forall i, \forall t, \quad t > 1 \] (14)

Set of constraints (14), which is written for all product types and all periods of planning, guarantee the satisfaction of safety stock of finished-products in working shifts. Set of constraints (15) represents the total production of non-defected products plus the inventory of finished-product in previous period should be greater than or equal to demand of the finished-product in current period.

Machine capacity Constraints

\[ \sum_{i=1}^{I} MH_{it} \cdot X_{iqt} \leq \overline{M}mc_{qt} - MC_i \cdot \overline{M}mc_{qt}, \quad \forall t, \quad q = 1 \] (16)

\[ \sum_{i=1}^{I} MH_{it} \cdot X_{iqt} \leq MCo \cdot \overline{M}mc_{qt} - MCri \cdot MCo \cdot \overline{M}mc_{qt}, \quad \forall t, \quad q = 2 \] (17)

Constraints (16) and (17) pledge that in regular time and overtime, the machine capacity is assured.

Warehouse Capacity Constraint

\[ \sum_{w=1}^{W} XP_{iwt} \leq \sum_{w=1}^{W} WhcP_{wit}, \quad \forall i, \forall t, \] (18)

\[ \sum_{m=1}^{M} \sum_{w=1}^{W} XR_{mwt} \leq \sum_{w=1}^{W} \sum_{m=1}^{M} WhcR_{mwt}, \quad \forall t, \] (19)

\[ \sum_{w=1}^{W} WhcP_{wit} + \sum_{w=1}^{W} WhcR_{mwt} \leq \overline{MW}Whm, \quad \forall i, \forall t, \] (20)

The first two constraints (18) and (19) gives the restrictions of actual inventories of finished products and raw materials. While (20) guarantees that each warehouse at each period will not be able to allow storage capacity of products an raw materials beyond its maximum warehouse available space.

Backorder, Budget limit and Non-negativity Constraints

There is backorder obeying the following:

\[ \sum_{w=1}^{W} B_{it} \leq \sum_{w=1}^{W} AsP_{it} \cdot DoP_{it}, \quad \forall i, \quad t \neq T \] (21)

\[ B_{iT} = 0, \quad \forall i \] (22)
\[
ToCo \leq \sum_{t=1}^{T} BUL_t
\]

Constraints (21) represent the backorder level at the end of period \(t\) cannot exceed the certain percent-age of the demand which determines the upper limit of shortage. While (22) assure that there is no possibility for backordering at the end of time horizon or last period.

A restriction on the available budget for each planning period is shown using (23), which ensures that the Total Cost (i.e., Eq. (1)) cannot go beyond the predetermined budget for the time horizon.

(24) and (25) both present non-negativity requirements on decision variables.

**Formation of Fuzzy Demand**

Fuzzy numbers like triangular and trapezoidal fuzzy numbers, can be used to represent demand in order to reflect this informational ambiguity. TFNs are used in this study to represent demand-related fuzzy data. Assuming the TFN of Demand is \(\overline{DoP}_{it} = (\overline{DoP}_1, \overline{DoP}_2, \overline{DoP}_3)\), in which \(\overline{DoP}_1\) is the most possible demand that certainly belongs to the set of available values (with a membership value of 1 after it is normalized), see figure 1 below. The lower bound value \(\overline{DoP}_2\) is the most pessimistic demand that has a small likelihood to belong to the set of available values (with a membership value of zero if normalized) and the upper bound value \(\overline{DoP}_3\) is the most optimistic demand with a small likelihood to belong to the set of available values (with a membership value of zero if normalized).

Let \(\mu(\overline{DoP}_{it})\) represent the arbitrary measurement of fuzzy demand in view of the Decision-maker, i.e. membership function, that defines the degree of \(x\) in the fuzzy space \(\overline{DoP}_{it}\) and figure 1 depicts the relationships of this function.

As seen in Figure 1 the membership function of fuzzy demand may be expressed as follows:

\[
\mu(\overline{DoP}_{it}) = \begin{cases} 
0 & \text{if } \overline{DoP}_it \leq \overline{DoP}_1 \\
\frac{(\overline{DoP}_it - \overline{DoP}_2)/(\overline{DoP}_3 - \overline{DoP}_1)}{1} & \text{if } \overline{DoP}_1 \leq \overline{DoP}_it \leq \overline{DoP}_2 \\
\frac{(\overline{DoP}_3 - \overline{DoP}_it)/(\overline{DoP}_3 - \overline{DoP}_2)}{1} & \text{if } \overline{DoP}_2 \leq \overline{DoP}_it \leq \overline{DoP}_3 \\
1 & \text{Otherwise}
\end{cases}
\]  

(26)
Supposing the decision-maker desires that APP meets the market demand for product $i$ in period $t$ with a possibility level. Using the fuzzy demand information, the constraint equations (11), (15) and (21) will be replaced with the following equations (27 to 29):

$$XP_{iw(t)} = XP_{iw(t-1)} + \sum_{q \in \{1,2\}} X_{iqt} - B_{it} - D_{it}, \quad \forall i, \forall w, \; t > 1$$ \hspace{1cm} (27)

$$D_{it} \leq \left(1 - \frac{Dr_{it}}{\beta_{it}}\right) \sum_{q \in \{1,2\}} X_{iqt} + XP_{i(t-1)}, \quad \forall i, \forall t, \; t > 1$$ \hspace{1cm} (28)

$$\sum_{w=1}^{W} B_{it} \leq \sum_{w=1}^{W} AsP_{it} \cdot D_{it} \quad \forall i, \; t \neq T$$ \hspace{1cm} (29)

Based on the ranking method developed by Jim'enez(1996), all fuzzy (imprecise) demand constraints in the model are translated to their corresponding crisp constraints as follows:

$$XP_{iw(t)} = XP_{iw(t-1)} + \sum_{q \in \{1,2\}} X_{iqt} - B_{it} - \left(\alpha \frac{D_{it}^1 + D_{it}^2}{2} + (1 - \alpha) \frac{D_{it}^2 + D_{it}^3}{2}\right), \forall i, w, t > 1$$ \hspace{1cm} (30)

$$\left(\alpha \frac{D_{it}^1 + D_{it}^2}{2} + (1 - \alpha) \frac{D_{it}^2 + D_{it}^3}{2}\right) \leq \left(1 - \frac{Dr_{it}}{\beta_{it}}\right) \sum_{q \in \{1,2\}} X_{iqt} + XP_{i(t-1)}, \forall i, t, \; t > 1$$ \hspace{1cm} (31)

$$\sum_{w=1}^{W} B_{it} \leq \sum_{w=1}^{W} AsP_{it} \cdot \left(\alpha \frac{D_{it}^1 + D_{it}^2}{2} + (1 - \alpha) \frac{D_{it}^2 + D_{it}^3}{2}\right) \quad \forall i, \; t \neq T$$ \hspace{1cm} (32)

The feasibility degree of the constraints that has been assigned by the DM based on the risk that he accepts on the violation of constraints imposed by the gotten solution is given by various values of $\alpha$.

**Possibilistic Programming on APP Problem with Imprecise Costs**

The Net-Profit APP decision problem that has already been looked at may be summarized as follows. Suppose that over a planning horizon of $T$, a corporation produces $N$ various products to satisfy market demand. On a medium time horizon, the environmental coefficients and associated parameters are often uncertain. As a result, across the planning horizon, related operational expenses and labor are imprecise. When dealing with such ambiguous APP decision problems, assigning a set of precise values for the environmental coefficients and associated parameters is problematic. The Net Profit APP objective function (4) can be restated as:

$$Z = \sum_{i=1}^{I} \sum_{t=1}^{T} S_{it} D_{it} - \sum_{i=1}^{I} \sum_{t=1}^{T} S_{it} B_{it} - \sum_{m=1}^{M} \sum_{w=1}^{W} \sum_{t=1}^{T} C_{mwt} X_{iwt} - \left[ \sum_{k=1}^{K} \sum_{t=1}^{T} C_{Wt} X_{Wt} \right]$$

$$+ \sum_{k=1}^{K} \sum_{t=1}^{T} C_{Ht} X_{Ht} + \sum_{k=1}^{K} \sum_{t=1}^{T} C_{Ft} X_{Ft} + \sum_{i=1}^{I} \sum_{t=1}^{T} \sum_{k=1}^{K} C_{it} X_{it} + \sum_{i=1}^{I} \sum_{t=1}^{T} C_{it} X_{it} + \sum_{i=1}^{I} \sum_{t=1}^{T} C_{it} X_{it}$$ \hspace{1cm} (33)
This work uses Wang and Liang's (2005) Possibility Linear Programming (PLP) technique to solve the APP problem with uncertainty. Fortunately, possibility distribution offers a useful substitute for dealing with underlying confusing phenomena when assessing environmental coefficients and associated factors (Zadeh, 1978; Inuiguchi and Sakawa, 1996; Hsu and Wang, 2001). This is by adopting the triangular Fuzzy number (TFN) to the APP problem under Fuzzy operational expenses.

**Approach to Resolving the Imprecise Objective Function**

The imprecise objective function of the Net-Profit Possibility APP programming model in the preceding section has a triangular possibility distribution. Geometrically, this imprecise objective is fully defined by three corner points: \((Z^p, 0), (Z^m, 1)\) and \((Z^o, 0)\). The imprecise objective can be maximized by pushing the three corner points towards the right. Because of the vertical coordinates of the critical points being fixed at either 1 or 0, the three horizontal coordinates are the only considerations. The new problem will be to solve:

Maximize \((Z^p, Z^m, Z^o)\)  

where \((Z^p, Z^m, Z^o)\) is the vector of the objective functions \(Z^p\), \(Z^m\) and \(Z^o\). It is important to make a minor change in order to maintain the possibility distribution's triangular shape (normal and convex). Instead of concurrently maximizing these three objectives, the new approach will maximize \(Z^m\), minimize \((Z^m - Z^p)\) and maximize \((Z^o - Z^m)\), where the first objective function \(Z^m\), is the basis of the last two objective functions, which are actually relative measures from it (see Figure 2). The three new objectives also support the earlier claim that doing so would shift the triangular possibility distribution towards the right.

This suggested approach equates to maximizing the most possible value of the imprecise profit (at the point of possibility degree = 1). At the same time, it has minimized the inferior side of the possibility distribution. This means minimizing the region (I), which in the perspective is similar to "the danger of receiving reduced profit." Also, this has increased "the chance of generating larger profit," which is similar to area (II) of the probability distribution. Similar to Figure 2, it would be preferred to have the possibility distribution of B against that of A. thus the auxiliary problem of equation (27) then result to three brand-new, precise objective functions as shown below;
Max $Z_1 = Z^m$

$$= \sum_{i=1}^{I} \sum_{t=1}^{T} SRe_i DoPi_t - \sum_{i=1}^{I} \sum_{t=1}^{T} SRe_i Bi_t - \sum_{m=1}^{W} \sum_{w=1}^{W} \sum_{t=1}^{T} CoM_{mwt}^m X_{iqt} - \sum_{k=1}^{K} \sum_{t=1}^{T} CoW_{k}^m X_{Wkt}$$

$$+ \sum_{k=1}^{K} \sum_{t=1}^{T} CoH_{k}^m X_{Hkt} + \sum_{k=1}^{K} \sum_{t=1}^{T} CoF_{k}^m X_{Fkt} + \sum_{l=1}^{I} \sum_{w=1}^{W} \sum_{t=1}^{T} Coh_{lwt}^m X_{lwt}$$

$$+ \sum_{m=1}^{M} \sum_{w=1}^{W} \sum_{t=1}^{T} CoR_{mwt}^m X_{mwt} + \sum_{l=1}^{I} \sum_{q \in [1,2]} \sum_{t=1}^{T} CoO_{iq}^m X_{iqt}$$

$$= \sum_{i=1}^{I} \sum_{t=1}^{T} SRe_i DoPi_t - \sum_{i=1}^{I} \sum_{t=1}^{T} SRe_i Bi_t - \sum_{m=1}^{W} \sum_{w=1}^{W} \sum_{t=1}^{T} (CoM_{mwt}^m - CoM_{mwt}^p) X_{iqt} - \sum_{k=1}^{K} \sum_{t=1}^{T} (CoW_{k}^m - CoW_{k}^p) X_{Wkt}$$

$$+ \sum_{k=1}^{K} \sum_{t=1}^{T} (CoH_{k}^m - CoH_{k}^p) X_{Hkt} + \sum_{k=1}^{K} \sum_{t=1}^{T} (CoF_{k}^m - CoF_{k}^p) X_{Fkt} + \sum_{l=1}^{I} \sum_{w=1}^{W} \sum_{t=1}^{T} (Coh_{lwt}^m - Coh_{lwt}^p) X_{lwt}$$

$$+ \sum_{m=1}^{M} \sum_{w=1}^{W} \sum_{t=1}^{T} (CoR_{mwt}^m - CoR_{mwt}^p) X_{mwt} + \sum_{l=1}^{I} \sum_{q \in [1,2]} \sum_{t=1}^{T} (CoO_{iq}^m - CoO_{iq}^p) X_{iqt}$$

Min $Z_2 = (Z^m - Z^p)$

$$= \sum_{i=1}^{I} \sum_{t=1}^{T} SRe_i DoPi_t - \sum_{i=1}^{I} \sum_{t=1}^{T} SRe_i Bi_t - \sum_{m=1}^{W} \sum_{w=1}^{W} \sum_{t=1}^{T} (CoM_{mwt}^p - CoM_{mwt}^m) X_{iqt} - \sum_{k=1}^{K} \sum_{t=1}^{T} (CoW_{k}^p - CoW_{k}^m) X_{Wkt}$$

$$+ \sum_{k=1}^{K} \sum_{t=1}^{T} (CoH_{k}^p - CoH_{k}^m) X_{Hkt} + \sum_{k=1}^{K} \sum_{t=1}^{T} (CoF_{k}^p - CoF_{k}^m) X_{Fkt} + \sum_{l=1}^{I} \sum_{w=1}^{W} \sum_{t=1}^{T} (Coh_{lwt}^p - Coh_{lwt}^m) X_{lwt}$$

$$+ \sum_{m=1}^{M} \sum_{w=1}^{W} \sum_{t=1}^{T} (CoR_{mwt}^m - CoR_{mwt}^p) X_{mwt} + \sum_{l=1}^{I} \sum_{q \in [1,2]} \sum_{t=1}^{T} (CoO_{iq}^p - CoO_{iq}^m) X_{iqt}$$

Max $Z_3 = (Z^o - Z^m)$

$$= \sum_{i=1}^{I} \sum_{t=1}^{T} SRe_i DoPi_t - \sum_{i=1}^{I} \sum_{t=1}^{T} SRe_i Bi_t - \sum_{m=1}^{W} \sum_{w=1}^{W} \sum_{t=1}^{T} (CoM_{mwt}^m - CoM_{mwt}^o) X_{iqt} - \sum_{k=1}^{K} \sum_{t=1}^{T} (CoW_{k}^m - CoW_{k}^o) X_{Wkt}$$

$$+ \sum_{k=1}^{K} \sum_{t=1}^{T} (CoH_{k}^m - CoH_{k}^o) X_{Hkt} + \sum_{k=1}^{K} \sum_{t=1}^{T} (CoF_{k}^m - CoF_{k}^o) X_{Fkt} + \sum_{l=1}^{I} \sum_{w=1}^{W} \sum_{t=1}^{T} (Coh_{lwt}^m - Coh_{lwt}^o) X_{lwt}$$

$$+ \sum_{m=1}^{M} \sum_{w=1}^{W} \sum_{t=1}^{T} (CoR_{mwt}^m - CoR_{mwt}^o) X_{mwt} + \sum_{l=1}^{I} \sum_{q \in [1,2]} \sum_{t=1}^{T} (CoO_{iq}^m - CoO_{iq}^o) X_{iqt}$$

Also, the fuzzy decision-making of Bellman and Zadeh (1970) and Zimmermann’s fuzzy programming (1978) approach may be used to transform the auxiliary MOLP issue into an analogous single-goal LP problem. The three objective functions' Positive Ideal Solutions (PIS) and Negative Ideal Solutions (NIS) can be correspondingly described as follows.

$$Z_{1}^{PIS} = \text{Max} Z^m; \quad Z_{1}^{NIS} = \text{Min} Z^m$$

$$Z_{2}^{PIS} = \text{Min}(Z^m - Z^p); \quad Z_{2}^{NIS} = \text{Max}(Z^m - Z^p) (v^*)$$

$$Z_{3}^{PIS} = \text{Max}(Z^o - Z^m); \quad Z_{3}^{NIS} = \text{Min}(Z^o - Z^m)$$

**Fuzzy Multi-objective Goal Programming Development**

In classic models of GP, the decision maker has to specify a precise aspiration level (goal) for each of the objectives. In general, especially in large-scale problems, this is a very difficult task, and the use of the Fuzzy Set theory in GP models can overcome such problem, allowing decision makers to work with imprecise aspiration levels (Yaghoobi and Tamiz, 2007). In multiobjective programming, In fuzzifying the inequality signs; “=” “≤” and “≥”, Zimmermann (1978) used the symbol “~”, they are to be understood as “essentially greater than or equal to” and “essentially less than or equal to”, if an imprecise aspiration level is introduced to each of the objective functions then these fuzzy objectives are termed as fuzzy goals. Let $g_k$ be the aspiration level assigned to the kth objective $Z_k(x)$. Then the fuzzy goals are:

$$Z_k(x) \gtrless g_k$$  [for maximizing $Z_k(x)$]  and  $$Z_k(x) \lessgtr g_k$$  [for minimizing $Z_k(x)$]
In solving the problem, a general form of FGP model is considered:

\[
\text{find } \mathbf{x} \\
\text{to satisfy;}
\]

\[
\begin{align*}
Z_k(x) &\geq g_k & k = 1 \ldots n \\
Z_k(x) &\leq g_k & k = n + 1 \ldots J
\end{align*}
\]

subject to

\[
AX \begin{cases}
\leq b \\
\geq b
\end{cases}
\]

\[
X \geq 0
\]

(38)

For this paper, a FGP is employed in solving any of the APP models, like (4)–(25). Being able to use FGP approach with fuzzy goals, the aspiration levels should be calculated. Payoff table is used when the decision maker has no enough view point to determine the aspiration levels. Zimmermann (1978) used a Payoff table to develop an upper and lower limit that was used to formulate the membership functions of the fuzzy goals.

In the general form (38), the purpose of FGP is to find compromise solution \( \mathbf{X} \) such that all fuzzy goals are satisfied. \( g_k \) is the aspiration level for \( k \)th goal, \( AX \leq b \) are system constraints in vector notation. \( Z_k(x) \geq g_k \) Means that the \( k \)th fuzzy goal is approximately less than or equal to the aspiration level \( g_k \), and \( Z_k(x) \geq g_k \) Means that the \( k \)th fuzzy goal is approximately greater than or equal to the aspiration level \( g_k \) (Hannan, 1981).

The fuzzy decision-making concept of Bellman and Zadeh (1970) can be used to solve the planned multi-objective APP problem (4)–(25). Linear membership functions as proposed by Zimmermann (1978) are used to represent the fuzzy goals of decision makers.

The corresponding linear membership function for each objective function is defined (see figure 3) by;

\[
\mu(Z_1(x)) = \begin{cases} 
1 & Z_1 \leq Z_1^{PIS}, \\
\frac{Z_1 - Z_1^{NIS}}{Z_1^{PIS} - Z_1^{NIS}} & Z_1^{NIS} \leq Z_1 \leq Z_1^{PIS}, \\
0 & Z_1 \geq Z_1^{PIS},
\end{cases}
\]

\[
\mu(Z_2(x)) = \begin{cases} 
1 & Z_2 \leq Z_2^{PIS}, \\
\frac{Z_2^{NIS} - Z_2}{Z_2^{PIS} - Z_2^{NIS}} & Z_2^{NIS} \leq Z_2 \leq Z_2^{PIS}, \\
0 & Z_2 \geq Z_2^{PIS},
\end{cases}
\]

and \( \mu(Z_1(x)) \) and \( \mu(Z_3(x)) \) are similar.
Lastly, the APP is solved following equivalent single-objective linear programming model. Hence, the associated FGP model with fuzzy cost for the Net-Profit APP problem (4)-(25) is formulate as follows:

\[
\begin{align*}
\text{find} & \quad x \\
\text{Maximize} & \quad \mu(Z_k(x)) \\
\text{to satisfy; } & \\
\mu(Z_1(x)) &= \frac{Z_1 - Z_1^{\text{NIS}}}{Z_1^{\text{PIS}} - Z_1^{\text{NIS}}} \\
\mu(Z_2(x)) &= \frac{Z_2^{\text{NIS}} - Z_2}{Z_2^{\text{PIS}} - Z_2^{\text{NIS}}} \\
\mu(Z_3(x)) &= \frac{Z_3 - Z_3^{\text{NIS}}}{Z_3^{\text{PIS}} - Z_3^{\text{NIS}}} \\
\mu(Z_j(x)) &\in [0,1], \quad j = 1,2,3 \\
\end{align*}
\]

(41)

**Model Implementation**

**Case description**

The case study of Rich Pharmaceuticals Limited (RPL) was utilized to show how useful the suggested methodology is. RPL is one of the leading producers of pharmaceuticals in Nigeria. RPL's goods are mostly sold in Southern and Middle belt of Nigeria, some parts of West and East Africa, they have recently experienced strong demand. RPL must monitor financial data and assess performance if it is to expand its company. The company's net profit margin is one statistic they have to pay attention to. RPL's business APP approach is to consider a dynamic labor force level over the planning horizon, allowing for the flexible meeting of demand through the use of inventories, overtime, and backorders.

Alternately, the DM can use a mathematical programming technique to create an aggregate production schedule for RPL factory. Based on company reports, the planning horizon spans for six months, May to October. The model includes two types of standard products. Production expenses for overtime are capped at 30% of production expenses for regular hours. Additionally, it is assumed that each product has no beginning inventory and no backorders at the last period. The inventory's maximum allowed storage area is 3000 m$^3$. In a day, there are two working shifts. 8 hours are allotted for regular production per shift, while 3 hours allotted for

---

**Figure 3. Linear Membership form**

- $\mu(Z_1(x))$
- $Z_1(x)$
- $Z_1^{\text{PIS}}$
- $Z_1^{\text{NIS}}$
- $\mu(Z_2(x))$
- $Z_2(x)$
- $Z_2^{\text{PIS}}$
- $Z_2^{\text{NIS}}$

Graph showing linear membership form between $Z_1(x)$ and $Z_2(x)$.
of overtime production. To produce these products, 10 types of raw materials are required. Repairs are done just in shift 2 (i.e., overtime) and the overall operating cost is as stated on Table 4 below. Table 5 gives the forecasted monthly demand for production, when demand for a certain period exceeds production capacity during regular hours and inventory levels are likewise insufficient to meet this demand, production is continued during overtime.

The purpose of the APP decision issue for the industrial instance that is addressed here is to develop a multiple fuzzy goals programming model for determining the optimal approach to adjust output rates, hiring and firing, inventory levels, overtime, and backorders in order to meet the targeted maximum profit using the Throughput accounting process. This APP choice is expected to reduce overall manufacturing costs, shorten the process, and increase sales and profit.

### Table 4. Related operating cost data

<table>
<thead>
<tr>
<th>Period</th>
<th>CoM (N/unit)</th>
<th>CoW (N/unit)</th>
<th>CoH (N/unit)</th>
<th>CoF (N/unit)</th>
<th>Coh (N/unit)</th>
<th>CoP (N/unit)</th>
<th>CoR (N/unit)</th>
<th>CoB (N/unit)</th>
<th>CoO (N/unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>45, 80, 140</td>
<td>35, 64, 100</td>
<td>10, 30, 55</td>
<td>15, 40, 60</td>
<td>0.5, 2, 4</td>
<td>800, 1400, 1600</td>
<td>1.5, 2, 4</td>
<td>5, 7, 11</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>45, 80, 135</td>
<td>35, 64, 100</td>
<td>10, 30, 55</td>
<td>15, 40, 60</td>
<td>0.5, 2, 4</td>
<td>850, 1400, 1600</td>
<td>1.5, 2, 4</td>
<td>5, 7, 10</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>48, 80, 140</td>
<td>35, 64, 100</td>
<td>10, 30, 55</td>
<td>15, 40, 60</td>
<td>0.5, 2, 4</td>
<td>800, 1400, 1500</td>
<td>1.2, 4.5</td>
<td>5.5, 7, 10</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>47, 80, 140</td>
<td>35, 64, 100</td>
<td>10, 30, 55</td>
<td>15, 40, 60</td>
<td>0.5, 2, 4</td>
<td>850, 1400, 1500</td>
<td>1.2, 4.5</td>
<td>5.5, 7, 11</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>47, 80, 145</td>
<td>35, 64, 100</td>
<td>10, 30, 55</td>
<td>15, 40, 60</td>
<td>0.5, 2, 4</td>
<td>850, 1400, 1500</td>
<td>1.5, 2, 4</td>
<td>5.5, 7, 11</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>47, 80, 140</td>
<td>35, 64, 100</td>
<td>10, 30, 55</td>
<td>15, 40, 60</td>
<td>0.5, 2, 4</td>
<td>800, 1400, 1600</td>
<td>1.5, 2, 4</td>
<td>5, 7, 10</td>
<td></td>
</tr>
</tbody>
</table>

### Table 5. Product Demand Forecasting in six months \( \text{DoP}_t = (\text{DoP}_{1t}, \text{DoP}_{2t}, \text{DoP}_{3t}) \) (unit/month)

<table>
<thead>
<tr>
<th>Product ( t )</th>
<th>Period ( t )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>(340,255,280)</td>
<td>(290,284,270)</td>
<td>(450,430,400)</td>
<td>(300,260,305)</td>
<td>(410,300,320)</td>
<td>(330,270,260)</td>
<td></td>
</tr>
</tbody>
</table>

### Results and Discussion

The following is a description of the RPL case’s solution process using the suggested APP-PP technique:

First Stage: Create the PP model for the APP choice issue in accordance with Equations (4) to (25).

Second Stage: Triangular possibility distributions are used to model the imprecise data as shown in Tables 4.

Third Stage: According to Equations (35) to (37) of the supplementary MOLP problem, create three new precise objective functions. In order to get the initial solutions for each of the objective functions, the original issue is solved using the standard single-objective LP method under the presumption that the DM provided the most likely value of the triangular distribution of each Fuzzy number as the precise value. LINGO 18.0 solver is used to solve the model. The objective values of the initial solutions using the model are \( \text{Max} Z_1 = 2770238 \), \( \text{Min} Z_2 = 1472820 \) and \( \text{Max} Z_2 = 3135912 \).

Fourth Stage: The PIS and NIS of the three new objective functions \( (Z_1, Z_1^{\text{NIS}}) = (2770238, 1114430) \) \( (Z_2, Z_2^{\text{NIS}}) = (1472820 ,2227640) \) and \( (Z_3, Z_3^{\text{NIS}}) = (3135912, 1658514) \). The fuzzy aspiration levels can be quantified using the linear and continuous membership function. According to Eq. (39) and (40), the relevant linear membership functions can be defined as shown below.
\[
\mu(Z_1(x)) = \begin{cases} 
1 & Z_1(x) \geq 1114430 \\
\frac{Z_1(x) - 1114430}{2770238 - 1114430} & 1114430 \leq Z_1(x) \leq 2770238 \\
0 & Z_1(x) \leq 1114430 
\end{cases}
\]

\[
\mu(Z_2(x)) = \begin{cases} 
1 & Z_2(x) \geq 2227640 \\
\frac{2227640 - Z_2(x)}{2227640 - 1472820} & 1472820 \leq Z_2(x) \leq 2227640 \\
0 & Z_2(x) \leq 1472820 
\end{cases}
\]

Equation (33), in addition, may be used to create the full equivalent single-objective LP model for the RPL situation.

Fifth Stage: Applying the FGP-APP gives the compromise solution as \( Z_1 = 2560411, Z_2 = 898850, Z_3 = 3049203 \). As will be determined by the triangular possibility distribution (see fig. 4), (₦1661561, ₦2560411, ₦5609614) is present in the improved profit as a result, and the overall degree of DM satisfaction is 0.873278.

Additional

In aggregate production planning, it is crucial to understand the output table that results from the interaction of a dynamic labor and an uncertain demand. The dynamic workforce's influence on labor utilization and production capacity adds another level of complication. Finding patterns between labor variations and production output in response to various demand levels requires analyzing the Table 6 by the DM. It necessitates evaluating how the firm can fulfill erratic
demand while reducing overages or shortages by adjusting workforce numbers. Insights from Table 6 also be used to inform choices on resource allocation, hiring practices, and inventory control measures, allowing the company to optimize their production plans in the face of changing labor markets and workforce dynamics.

Table 6. Operational Data (Abridged)

<table>
<thead>
<tr>
<th>Workforce</th>
<th>Z1</th>
<th>Z2</th>
<th>Z3</th>
<th>FGP</th>
</tr>
</thead>
<tbody>
<tr>
<td>XL11</td>
<td>92.55738</td>
<td>92.78828</td>
<td>92.55738</td>
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<tr>
<td>XL2</td>
<td>102.8415</td>
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</tr>
<tr>
<td>XL3</td>
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</tr>
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</tr>
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</tr>
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<tr>
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<table>
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Conclusions

In conclusion, the journal article "Possibilistic Aggregate Production Planning Considering Dynamic Workforce with Fuzzy Demand" clarifies the complicated world of production planning by fusing the complexity of dynamic workforce fluctuations and fuzzy demand forecasting. The study emphasizes the necessity of flexibility in resource allocation to successfully traverse the obstacles given by variable worker capabilities and uncertain demand. The paper provides insightful information on minimizing production risks and maximizing operational effectiveness by using a possibilistic approach. However, this study serves as a first step toward a deeper comprehension of this complex dynamic. Future research in this area may focus further on the creation of sophisticated prediction models that combine labor dynamics, demand volatility, and other contextual elements to improve the accuracy of decisions. Investigating the integration of cutting-edge technologies like artificial intelligence and machine learning might also result in groundbreaking approaches to agile production planning in the face of flexible demand scenarios and a mobile workforce.

Effective production planning has become a key approach for firms to ensure efficient operations, satisfy consumer needs, and maximize resource usage in today's dynamic and competitive business environment. Aggregate Production Planning (APP), one of the many planning methods, stands out as a crucial strategy that enables companies to strike a careful balance between production levels and inventories while staying in line with market expectations. Aggregate production planning helps businesses deal with the difficulties of varying demand, unpredictability in the supply chain, and cost considerations by concentrating on the overall picture of production over a specific time horizon.

References


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